# 240AlgebraI-090915, Hours 2-3

September-14-09
9:13 AM

0. Go over http://katlas.math.toronto.edu/drorbn/index.php?title=09-240

1. The real numbers: a set $\mathbb{R}$ with two binary ops $+$ & $\times$ and two special elements 0 & 1 s.t.

R1 $a+b=b+a$ $ab=ba$

R2 Assoc.

R3 0, 1

R4 negatives & inverses

R5 Distributivity.

Much of algebra, though not all, follows:

Follows: $(a+b)(a-b)=a^2-b^2$

Doesn't follow: $\forall a\ \exists x$ s.t. $a=x^2$ or $a=-x^2$

2. Def A Field F

Examples 1. The reals $\mathbb{R}$.

2. The rationals $\mathbb{Q}$

3. The complex numbers $\mathbb{C}=\{a+b\sqrt{-1}\}$

4. 0, 1 with

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

| · | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

5. 0, 1, 2, 3, 4, 5, 6 with a funny def. of $+$, $\times$.

3. Thm 1. $a+b=c+b \Rightarrow a=c$

2. $a\cdot b=c\cdot b$, $b\neq 0 \Rightarrow a=c$

3. If $0'$ is like 0, then $0'=0$

4. If $1'$ is like 1, then $1'=1$

Proofs...

4. If $1'$ is like 1, Then $1' = 1$

5. If $a+b = 0 = a+b'$ then $b = b'$
(so we can define $-a$)

6. If $a \neq 1$ & $ab = 1 = ab' \Rightarrow b = b'$
(so we can define $a^{-1}$)

7. $-(-a) = a$, $(a^{-1})^{-1} = a$

def: subtraction $a-b$, division $a/b$ when $b \neq 0$.

8. $a \cdot 0 = 0$

9. There's no $0^{-1}$

That's hard!

10. $(-a) \cdot b = a \cdot (-b) = -(a \cdot b)$

11. $(-a)(-b) = a \cdot b$

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4. Def characteristic