

Pensieve header: Kauffman States for tangles.

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\2024-03"];
Once[<< KnotTheory`];
Cut[pd_PD] := Module[{n = Length[pd]},
  pd /. {X[2 n, i_, 1, j_] => X[2 n, i, 2 n + 1, j],
    X[i_, 1, j_, 2 n] => X[i, 2 n + 1, j, 2 n], X[i_, 2 n, j_, 1] => X[i, 2 n, j, 2 n + 1]}
]
```

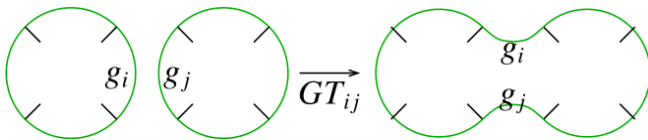
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.  
 Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:= CF[E_] := Expand[E];
```

```
In[*]:= SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}];
```

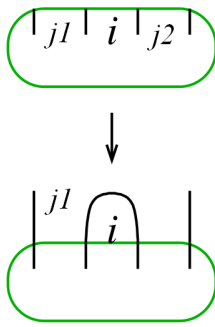
```
In[*]:= CF[Gb[f_]] := GCF[b][CF[f]]
```

```
In[*]:= Gb1[f1_] ⊕ Gb2[f2_] ^:= CF@GJoin[b1,b2][f1 f2];
```



GT for Gap Touch:

```
In[*]:= GTi,j@GB[{li___,i_,ri___},{lj___,j_,rj___},bs___][f_] := CF@GB[{ri,li,j,rj,lj,i},bs][f /. {
  Yi[l1___] Yj[l2___] => Sort@Yi[j, l1, l2] + Sort@Yj[i, l1, l2],
  Yi[l1___] Yk[ll___, j, rl___] => Sort@Yk[l1, ll, rl, i, j],
  Yi[l1___] Y[ll___, j, rl___] => Sort@Y[l1, ll, rl, i, j],
  Yj[l1___] Yk[ll___, i, rl___] => Sort@Yk[l1, ll, rl, i, j],
  Yj[l1___] Y[ll___, i, rl___] => Sort@Y[l1, ll, rl, i, j],
  Y[___, i, ___] Y[___, j, ___] -> 0,
  Y[___, i, ___] Y[___, j, ___] -> 0,
  Y[___, j, ___] Y[___, i, ___] -> 0,
  Y[___, i, ___] Y[___, j, ___] -> 0
}]
```



cor·don  (kôr'dn)



n.

1. A line of people, military posts, or ships stationed around an area to enclose or guard it: *a police cordon*.
2. A rope, line, tape, or similar border stretched around an area, usually by the police, indicating that access is restricted.

In[\*]:=

```

Cordoni@GB[{li___,i,ri___},bs___][f_] := Module[{j1, j2},
  {j1, j2} = {First@{ri, li}, Last@{ri, li}};
  CF@GB[Most@{ri, li},bs][f /. {
    Yi[L___] → 0,
    Yk[LL___, i, rL___] ⇒ Yk[LL, rL],
    Y[LL___, i, rL___] ⇒ Y[LL, rL]
  } /. {
    Yj1[L1___] Yj2[L2___] ⇒ Sort@Yj1[L1, L2],
    Yj1[___, j2, ___] → 0,
    Yj2[___, j1, ___] → 0,
    Yj1[Ls___] Yk[LL___, j2, rL___] ⇒ Sort@Yk[j1, Ls, LL, rL],
    Yj2[Ls___] Yk[LL___, j1, rL___] ⇒ Sort@Yk[j1, Ls, LL, rL],
    Yj1[Ls___] Y[LL___, j2, rL___] ⇒ Sort@Y[j1, Ls, LL, rL],
    Yj2[Ls___] Y[LL___, j1, rL___] ⇒ Sort@Y[j1, Ls, LL, rL],
    Y[___, j1, ___] Y[___, j2, ___] → 0,
    Y[___, j2, ___] Y[___, j1, ___] → 0,
    Y[___, j1, ___] Y[___, j2, ___] → 0,
    Y[___, j1, ___, j2, ___] → 0,
    Y[___, j2, ___, j1, ___] → 0,
    Y[___, j1, ___, j2, ___] → 0,
    Y[___, j2, ___, j1, ___] → 0
  }
]
    
```

Strand Operations. c for contract, mc for magnetic contract:

In[\*]:=

```

ci,j@t : GB[{li___,i,ri___},{___,j,___},___][___] := t // GTj,First@{ri,li} // Cordonj
    
```

In[\*]:=

```

ci,j@t : GB[{___,i,j,___},___][___] := Cordonj@t
ci,j@t : GB[{j,___,i},___][___] := Cordonj@t
ci,j@t : GB[{___,j,i,___},___][___] := Cordoni@t
ci,j@t : GB[{i,___,j},___][___] := Cordoni@t
    
```

```
In[*]:= mc[E_] := E /.
  t : GB[{{_,i_,_},{_,j_,_},_][_] | GB[{{_,i_,j_,_},_][_] | GB[{{j_,_,i_,_},_][_] /;
    i + j == 0 => ci,j@t
```

“KSI” for Kauffman States Invariant.

```
In[*]:= KSI@Pi,j := CF@GB[{{i,j}}][Yi Yj];
KSI[x : X[i_, j_, k_, l_]] := KSI@If[PositiveQ[x], X-i,j,k,-l, X̄-j,k,l,-i];
KSI[Xi,j,k,l] :=
  CF@GB[{{i,j,k,l}}][m T-1 Yk[i] Yj Yl + T Yi[k] Yj Yl + Yj[l] Yi Yk + Yl[j] Yi Yk];
KSI[X̄i,j,k,l] :=
  CF@GB[{{i,j,k,l}}][m T Yk[i] Yj Yl + T-1 Yi[k] Yj Yl + Yj[l] Yi Yk + Yl[j] Yi Yk];
KSI[K_] := Fold[mc[#1 ⊕ #2] &, GB[1], List@@(KSI /@ PD@K)];
```

## Knots

```
In[*]:= KSIK[K_] := KSI[Cut@PD@K][1] /. {Y_[] -> 1, T -> T1/2, m -> -1}
```

```
In[*]:= KSIK[Knot[8, 17]]
```

Out[\*]=

$$11 - \frac{1}{T^3} + \frac{4}{T^2} - \frac{8}{T} - 8T + 4T^2 - T^3$$

```
In[*]:= Alexander[Knot[8, 17]][T]
```

Out[\*]=

$$11 - \frac{1}{T^3} + \frac{4}{T^2} - \frac{8}{T} - 8T + 4T^2 - T^3$$

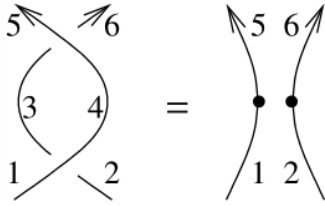
```
In[*]:= Monitor[
  Timing@Table[res = (K -> (KSIK[K] / Alexander[K][T])), {K, AllKnots[{3, 10}]},
  res]
```

Out[\*]=

{1442.59, {Knot[3, 1] -> 1, Knot[4, 1] -> 1, Knot[5, 1] -> 1, Knot[5, 2] -> 1, Knot[6, 1] -> 1, Knot[6, 2] -> 1, Knot[6, 3] -> 1, Knot[7, 1] -> 1, Knot[7, 2] -> 1, Knot[7, 3] -> 1, Knot[7, 4] -> 1, Knot[7, 5] -> 1, Knot[7, 6] -> 1, Knot[7, 7] -> 1, Knot[8, 1] -> 1, Knot[8, 2] -> 1, Knot[8, 3] -> 1, Knot[8, 4] -> 1, Knot[8, 5] -> 1, Knot[8, 6] -> 1, Knot[8, 7] -> 1, Knot[8, 8] -> 1, Knot[8, 9] -> 1, Knot[8, 10] -> 1, Knot[8, 11] -> 1, Knot[8, 12] -> 1, Knot[8, 13] -> 1, Knot[8, 14] -> 1, Knot[8, 15] -> 1, Knot[8, 16] -> 1, Knot[8, 17] -> 1, Knot[8, 18] -> 1, Knot[8, 19] -> 1, Knot[8, 20] -> 1, Knot[8, 21] -> 1, Knot[9, 1] -> 1, Knot[9, 2] -> 1, Knot[9, 3] -> 1, Knot[9, 4] -> 1, Knot[9, 5] -> 1, Knot[9, 6] -> 1, Knot[9, 7] -> 1, Knot[9, 8] -> 1, Knot[9, 9] -> 1, Knot[9, 10] -> 1, Knot[9, 11] -> 1, Knot[9, 12] -> 1, Knot[9, 13] -> 1, Knot[9, 14] -> 1, Knot[9, 15] -> 1, Knot[9, 16] -> 1, Knot[9, 17] -> 1, Knot[9, 18] -> 1, Knot[9, 19] -> 1, Knot[9, 20] -> 1,

Knot [9, 21] → 1, Knot [9, 22] → 1, Knot [9, 23] → 1, Knot [9, 24] → 1, Knot [9, 25] → 1,  
 Knot [9, 26] → 1, Knot [9, 27] → 1, Knot [9, 28] → 1, Knot [9, 29] → 1, Knot [9, 30] → 1,  
 Knot [9, 31] → 1, Knot [9, 32] → 1, Knot [9, 33] → 1, Knot [9, 34] → 1, Knot [9, 35] → 1,  
 Knot [9, 36] → 1, Knot [9, 37] → 1, Knot [9, 38] → 1, Knot [9, 39] → 1, Knot [9, 40] → 1,  
 Knot [9, 41] → 1, Knot [9, 42] → 1, Knot [9, 43] → 1, Knot [9, 44] → 1, Knot [9, 45] → 1,  
 Knot [9, 46] → 1, Knot [9, 47] → 1, Knot [9, 48] → 1, Knot [9, 49] → 1, Knot [10, 1] → 1,  
 Knot [10, 2] → 1, Knot [10, 3] → 1, Knot [10, 4] → 1, Knot [10, 5] → 1, Knot [10, 6] → 1,  
 Knot [10, 7] → 1, Knot [10, 8] → 1, Knot [10, 9] → 1, Knot [10, 10] → 1, Knot [10, 11] → 1,  
 Knot [10, 12] → 1, Knot [10, 13] → 1, Knot [10, 14] → 1, Knot [10, 15] → 1, Knot [10, 16] → 1,  
 Knot [10, 17] → 1, Knot [10, 18] → 1, Knot [10, 19] → 1, Knot [10, 20] → 1, Knot [10, 21] → 1,  
 Knot [10, 22] → 1, Knot [10, 23] → 1, Knot [10, 24] → 1, Knot [10, 25] → 1, Knot [10, 26] → 1,  
 Knot [10, 27] → 1, Knot [10, 28] → 1, Knot [10, 29] → 1, Knot [10, 30] → 1, Knot [10, 31] → 1,  
 Knot [10, 32] → 1, Knot [10, 33] → 1, Knot [10, 34] → 1, Knot [10, 35] → 1, Knot [10, 36] → 1,  
 Knot [10, 37] → 1, Knot [10, 38] → 1, Knot [10, 39] → 1, Knot [10, 40] → 1, Knot [10, 41] → 1,  
 Knot [10, 42] → 1, Knot [10, 43] → 1, Knot [10, 44] → 1, Knot [10, 45] → 1, Knot [10, 46] → 1,  
 Knot [10, 47] → 1, Knot [10, 48] → 1, Knot [10, 49] → 1, Knot [10, 50] → 1, Knot [10, 51] → 1,  
 Knot [10, 52] → 1, Knot [10, 53] → 1, Knot [10, 54] → 1, Knot [10, 55] → 1, Knot [10, 56] → 1,  
 Knot [10, 57] → 1, Knot [10, 58] → 1, Knot [10, 59] → 1, Knot [10, 60] → 1, Knot [10, 61] → 1,  
 Knot [10, 62] → 1, Knot [10, 63] → 1, Knot [10, 64] → 1, Knot [10, 65] → 1, Knot [10, 66] → 1,  
 Knot [10, 67] → 1, Knot [10, 68] → 1, Knot [10, 69] → 1, Knot [10, 70] → 1, Knot [10, 71] → 1,  
 Knot [10, 72] → 1, Knot [10, 73] → 1, Knot [10, 74] → 1, Knot [10, 75] → 1, Knot [10, 76] → 1,  
 Knot [10, 77] → 1, Knot [10, 78] → 1, Knot [10, 79] → 1, Knot [10, 80] → 1, Knot [10, 81] → 1,  
 Knot [10, 82] → 1, Knot [10, 83] → 1, Knot [10, 84] → 1, Knot [10, 85] → 1, Knot [10, 86] → 1,  
 Knot [10, 87] → 1, Knot [10, 88] → 1, Knot [10, 89] → 1, Knot [10, 90] → 1, Knot [10, 91] → 1,  
 Knot [10, 92] → 1, Knot [10, 93] → 1, Knot [10, 94] → 1, Knot [10, 95] → 1, Knot [10, 96] → 1,  
 Knot [10, 97] → 1, Knot [10, 98] → 1, Knot [10, 99] → 1, Knot [10, 100] → 1, Knot [10, 101] → 1,  
 Knot [10, 102] → 1, Knot [10, 103] → 1, Knot [10, 104] → 1, Knot [10, 105] → 1,  
 Knot [10, 106] → 1, Knot [10, 107] → 1, Knot [10, 108] → 1, Knot [10, 109] → 1,  
 Knot [10, 110] → 1, Knot [10, 111] → 1, Knot [10, 112] → 1, Knot [10, 113] → 1,  
 Knot [10, 114] → 1, Knot [10, 115] → 1, Knot [10, 116] → 1, Knot [10, 117] → 1,  
 Knot [10, 118] → 1, Knot [10, 119] → 1, Knot [10, 120] → 1, Knot [10, 121] → 1,  
 Knot [10, 122] → 1, Knot [10, 123] → 1, Knot [10, 124] → 1, Knot [10, 125] → 1,  
 Knot [10, 126] → 1, Knot [10, 127] → 1, Knot [10, 128] → 1, Knot [10, 129] → 1,  
 Knot [10, 130] → 1, Knot [10, 131] → 1, Knot [10, 132] → 1, Knot [10, 133] → 1,  
 Knot [10, 134] → 1, Knot [10, 135] → 1, Knot [10, 136] → 1, Knot [10, 137] → 1,  
 Knot [10, 138] → 1, Knot [10, 139] → 1, Knot [10, 140] → 1, Knot [10, 141] → 1,  
 Knot [10, 142] → 1, Knot [10, 143] → 1, Knot [10, 144] → 1, Knot [10, 145] → 1,  
 Knot [10, 146] → 1, Knot [10, 147] → 1, Knot [10, 148] → 1, Knot [10, 149] → 1,  
 Knot [10, 150] → 1, Knot [10, 151] → 1, Knot [10, 152] → 1, Knot [10, 153] → 1,  
 Knot [10, 154] → 1, Knot [10, 155] → 1, Knot [10, 156] → 1, Knot [10, 157] → 1,  
 Knot [10, 158] → 1, Knot [10, 159] → 1, Knot [10, 160] → 1, Knot [10, 161] → 1,  
 Knot [10, 162] → 1, Knot [10, 163] → 1, Knot [10, 164] → 1, Knot [10, 165] → 1 }

## Reidemeister 2



In[\*]:= CF[KSI@PD[X<sub>-2,4,3,-1</sub>, X̄<sub>-4,6,5,-3</sub>] /. m → -1]

Out[\*]=

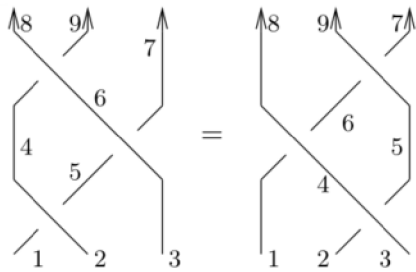
G<sub>B</sub>[{-2,6,5,-1}] [Y<sub>-2</sub>[5] Y<sub>-1</sub>[ ] Y<sub>6</sub>[ ] + Y<sub>-1</sub>[ ] Y<sub>5</sub>[-2] Y<sub>6</sub>[ ]]

In[\*]:= GT<sub>5,-2</sub>@KSI@PD[P<sub>-1,5</sub>, P<sub>-2,6</sub>]

Out[\*]=

G<sub>B</sub>[{-2,6,5,-1}] [Y<sub>-2</sub>[5] Y<sub>-1</sub>[ ] Y<sub>6</sub>[ ] + Y<sub>-1</sub>[ ] Y<sub>5</sub>[-2] Y<sub>6</sub>[ ]]

### Reidemeister 3



In[\*]:= lhs = KSI[PD[X<sub>-2,5,4,-1</sub>, X<sub>-3,7,6,-5</sub>, X<sub>-6,9,8,-4</sub>]] /. m → -1

Out[\*]=

G<sub>B</sub>[{-3,7,9,8,-1,-2}] [

$$\begin{aligned}
 & T^3 Y_{-3}[9] Y_{-2}[8] Y_{-1}[ ] Y_7[ ] + T Y_{-3}[ ] Y_{-2}[7] Y_{-1}[9] Y_8[ ] + T^2 Y_{-3}[-1, 9] Y_{-2}[ ] Y_7[ ] Y_8[ ] - \\
 & Y_{-2}[ ] Y_{-1}[-3, 9] Y_7[ ] Y_8[ ] - \frac{Y_{-3}[ ] Y_{-1}[9] Y_7[-2] Y_8[ ]}{T} + \frac{Y_{-3}[9] Y_{-1}[ ] Y_7[ ] Y_8[-2]}{T} + \\
 & T^2 Y_{-3}[ ] Y_{-2}[7, 8] Y_{-1}[ ] Y_9[ ] + T Y_{-3}[-1] Y_{-2}[ ] Y_7[8] Y_9[ ] + T Y_{-2}[ ] Y_{-1}[-3] Y_7[8] Y_9[ ] - \\
 & Y_{-3}[ ] Y_{-1}[ ] Y_7[-2, 8] Y_9[ ] - \frac{Y_{-3}[-1] Y_{-2}[ ] Y_8[7] Y_9[ ]}{T} - \frac{Y_{-2}[ ] Y_{-1}[-3] Y_8[7] Y_9[ ]}{T} + \\
 & \frac{Y_{-3}[ ] Y_{-1}[ ] Y_8[-2, 7] Y_9[ ]}{T^2} - T Y_{-2}[8] Y_{-1}[ ] Y_7[ ] Y_9[-3] - \frac{Y_{-1}[ ] Y_7[ ] Y_8[-2] Y_9[-3]}{T^3} + \\
 & T Y_{-3}[ ] Y_{-2}[7] Y_8[ ] Y_9[-1] - \frac{Y_{-3}[ ] Y_7[-2] Y_8[ ] Y_9[-1]}{T} + \frac{Y_{-2}[ ] Y_7[ ] Y_8[ ] Y_9[-3, -1]}{T^2} ]
 \end{aligned}$$

In[\*]:= rhs = KSI[PD[X<sub>-3,5,4,-2</sub>, X<sub>-4,6,8,-1</sub>, X<sub>-5,7,9,-6</sub>]] /. m -> -1

Out[\*]=

$$G_{B\{-3,7,9,8,-1,-2\}} \left[ \begin{aligned} & T^3 Y_{-3}[9] Y_{-2}[8] Y_{-1}[] Y_7[] + T Y_{-3}[] Y_{-2}[7] Y_{-1}[9] Y_8[] + T^2 Y_{-3}[-1, 9] Y_{-2}[] Y_7[] Y_8[] - \\ & Y_{-2}[] Y_{-1}[-3, 9] Y_7[] Y_8[] + T Y_{-3}[] Y_{-1}[9] Y_7[-2] Y_8[] - T Y_{-3}[9] Y_{-1}[] Y_7[] Y_8[-2] + \\ & T^2 Y_{-3}[] Y_{-2}[7, 8] Y_{-1}[] Y_9[] + T Y_{-3}[-1] Y_{-2}[] Y_7[8] Y_9[] - \frac{Y_{-2}[] Y_{-1}[-3] Y_7[8] Y_9[]}{T} - \\ & Y_{-3}[] Y_{-1}[] Y_7[-2, 8] Y_9[] + T Y_{-3}[-1] Y_{-2}[] Y_8[7] Y_9[] - \frac{Y_{-2}[] Y_{-1}[-3] Y_8[7] Y_9[]}{T} + \\ & \frac{Y_{-3}[] Y_{-1}[] Y_8[-2, 7] Y_9[]}{T^2} + \frac{Y_{-2}[8] Y_{-1}[] Y_7[] Y_9[-3]}{T} - \frac{Y_{-1}[] Y_7[] Y_8[-2] Y_9[-3]}{T^3} - \\ & \frac{Y_{-3}[] Y_{-2}[7] Y_8[] Y_9[-1]}{T} - \frac{Y_{-3}[] Y_7[-2] Y_8[] Y_9[-1]}{T} + \frac{Y_{-2}[] Y_7[] Y_8[] Y_9[-3, -1]}{T^2} \end{aligned} \right]$$

In[\*]:= lhs[[1]] - rhs[[1]]

Out[\*]=

$$\begin{aligned} & - \frac{Y_{-3}[] Y_{-1}[9] Y_7[-2] Y_8[]}{T} - T Y_{-3}[] Y_{-1}[9] Y_7[-2] Y_8[] + \frac{Y_{-3}[9] Y_{-1}[] Y_7[] Y_8[-2]}{T} + \\ & T Y_{-3}[9] Y_{-1}[] Y_7[] Y_8[-2] + \frac{Y_{-2}[] Y_{-1}[-3] Y_7[8] Y_9[]}{T} + T Y_{-2}[] Y_{-1}[-3] Y_7[8] Y_9[] - \\ & \frac{Y_{-3}[-1] Y_{-2}[] Y_8[7] Y_9[]}{T} - T Y_{-3}[-1] Y_{-2}[] Y_8[7] Y_9[] - \frac{Y_{-2}[8] Y_{-1}[] Y_7[] Y_9[-3]}{T} - \\ & T Y_{-2}[8] Y_{-1}[] Y_7[] Y_9[-3] + \frac{Y_{-3}[] Y_{-2}[7] Y_8[] Y_9[-1]}{T} + T Y_{-3}[] Y_{-2}[7] Y_8[] Y_9[-1] \end{aligned}$$