

Pensieve Header: Testing the log-concavity of the Alexander polynomial; inspired by “Log-concavity of the Alexander polynomial”, arXiv:2303.04733, by Elena S. Hafner, Karola Mészáros, Alexander Vidinas. Continued for ρ_1 (fails) at pensieve://Projects/APAI/

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\2023-03"];
Once[<< KnotTheory`]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:= ConcaveQ[s_List] := And@@Thread[2 s[[2 ;; -2]] >= s[[1 ;; -3]] + s[[3 ;; -1]]];
```

```
In[*]:= alex = Alexander[Knot[8, 17]] [-T]
CoefficientList[T^Exponent[alex, T] alex, T]
ConcaveQ[Log[Abs@CoefficientList[T^Exponent[alex, T] alex, T]]]
```

Out[*]=

$$11 + \frac{1}{T^3} + \frac{4}{T^2} + \frac{8}{T} + 8T + 4T^2 + T^3$$

Out[*]=

```
{1, 4, 8, 11, 8, 4, 1}
```

Out[*]=

```
True
```

```
In[*]:= K = Knot[8, 19]
alex = Alexander[K] [-T]
cs = CoefficientList[Sign[alex /. T -> 1] T^Exponent[alex, T] alex, T]
Not[And@@(Positive/@cs)]
```

Out[*]=

```
Knot[8, 19]
```

Out[*]=

$$1 - \frac{1}{T^3} - \frac{1}{T^2} - T^2 - T^3$$

Out[*]=

```
{1, 1, 0, -1, 0, 1, 1}
```

Out[*]=

```
True
```

```
In[*]:= AlexanderLogConcaveQ[K_] := Module[{alex, T, cs},
alex = Alexander[K] [-T];
cs = CoefficientList[Sign[alex /. T -> 1] T^Exponent[alex, T] alex, T];
If[Not[And@@(Positive/@cs)], Return@False];
If[Length[cs] < 3, Return@True];
ConcaveQ[Log@cs]
]
```

```
In[*]:= NLCs = Select[AllKnots[{3, 10}], Not[AlexanderLogConcaveQ[#]] &]
Out[*]=
{Knot[8, 19], Knot[9, 42], Knot[9, 43], Knot[9, 47], Knot[10, 124], Knot[10, 125],
Knot[10, 128], Knot[10, 134], Knot[10, 138], Knot[10, 139], Knot[10, 142], Knot[10, 145],
Knot[10, 152], Knot[10, 153], Knot[10, 154], Knot[10, 160], Knot[10, 161]}

In[*]:= AlternatingQ /@ NLCs
Out[*]=
{False, False, False, False, False, False, False, False,
False, False, False, False, False, False, False, False}

In[*]:= Total[
  AlternatingQ /@ Select[AllKnots[{11, 14}], Not[AlexanderLogConcaveQ[#]] &]
]

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.
KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of
2005.
KnotTheory: Loading precomputed data in KnotTheory/12A.dts.
KnotTheory: Loading precomputed data in KnotTheory/12N.dts.
General: Further output of KnotTheory::loading will be suppressed during this calculation.

Out[*]=
6233 False
```