Extensions of representations

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$$\begin{tikzcd}
0 \arrow[r] & A \arrow[r, "\alpha"'] & B \arrow[r, "\beta"'] \arrow[loop below, "G"] & C \arrow[r] \arrow[l, bend right, "\gamma"'] \arrow[ll, dashed, bend right=40, "\delta"'] & 0
\end{tikzcd}$$

$B \cong \{(a,c)\}$ $\qquad\qquad$ $\varphi : G \times C \longrightarrow A$

$g(a,c) = (ga + \varphi_g c\,,\, gc)$ $\qquad$ $\varphi_g c \in A$

$g = g_1 g_2$: $\quad \varphi_{g_1 g_2}(c) = g_1 \varphi_{g_2}(c) + \varphi_{g_1}(g_2 c)$

$\Phi_\delta : B \longrightarrow B$ by $\quad (a,c) \longmapsto (a + \delta c, c)$

inverse : $(a,c) \longmapsto (a - \delta c\,,\, c)$

$\delta : C \longrightarrow A$

$$g \triangleright_\delta (a,c) := \Phi_\delta^{-1}(g \triangleright (\Phi_\delta(a,c)))$$

$$= \Phi_\delta^{-1}(g \triangleright (a + \delta c, c)) =$$

$$= \Phi_\delta^{-1}(ga + g\delta c + \varphi_g(c), gc)$$

$$= (ga + \underbrace{g\delta c + \varphi_g(c) - \delta g c}_{\varphi_g^\delta(c)}, gc)$$

In the CUW case:

$$\begin{tikzcd}
0 \arrow[r] & \underset{G \otimes G^*}{\{UW\}} \arrow[r] & \{C, UW\} \arrow[r] & \underset{R^n}{\{C\}} \arrow[r] & 0
\end{tikzcd}$$

So $\varphi_g(c)$ is $\varphi : PWB_n \times \underline{n} \longrightarrow G \otimes G^*$