

Analyzing Turbo-Gassner

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From TurboGassner.nb:

```
TGRule = {
  f_ . v_j => f v_j + f u_{k,j,k} + ((1 - t_j^{-1}) (t_j \partial_{t_j} f - t_k \partial_{t_k} f) - f) u_{j,j,k},
  f_ . v_k => f (1 - t_j) v_j + f t_j v_k - t_j^{-1} (t_j - 1)^2 (t_j \partial_{t_j} f - t_k \partial_{t_k} f) u_{j,j,k} +
    (t_j - 1) (t_j \partial_{t_j} f - t_k \partial_{t_k} f) u_{k,j,k},
  f_ . v_i => f v_i + (1 - t_j^{-1}) (t_j \partial_{t_j} f - t_k \partial_{t_k} f) u_{j,i,k},
  u_{j,j,j} => u_{j,j,j} + (1 - t_j^{-1}) u_{j,j,k},
  u_{j,j,k} => t_j^{-1} u_{j,j,k},
  u_{j,k,j} | u_{k,j,j} => (1 - t_j) u_{j,j,j} - t_j^{-1} (t_j - 1)^2 u_{j,j,k} + t_j u_{k,j,j} + (t_j - 1) u_{k,j,k},
  u_{j,k,k} | u_{k,j,k} => (t_j^{-1} - 1) u_{j,j,k} + u_{k,j,k},
  u_{k,k,j} => (t_j - 1)^2 u_{j,j,j} + t_j^{-1} (t_j - 1)^3 u_{j,j,k} - 2 (t_j - 1) t_j u_{k,j,j} -
    2 (t_j - 1)^2 u_{k,j,k} + t_j^2 u_{k,k,j} + (t_j - 1) t_j u_{k,k,k},
  u_{k,k,k} => t_j^{-1} (t_j - 1)^2 u_{j,j,k} + (2 - 2 t_j) u_{k,j,k} + t_j u_{k,k,k},
  u_{j,j,l} => u_{j,j,l},
  u_{j,k,l} | u_{k,j,l} => (1 - t_j) u_{j,j,l} + t_j u_{k,j,l},
  u_{k,k,l} => (-1 + t_j)^2 u_{j,j,l} - 2 (t_j - 1) t_j u_{k,j,l} + t_j^2 u_{k,k,l},
  u_{i,j,j} | u_{j,i,j} => u_{j,i,j} + (1 - t_j^{-1}) u_{j,i,k},
  u_{i,j,k} | u_{j,i,k} => t_j^{-1} u_{j,i,k},
  u_{i,k,j} | u_{k,i,j} => (1 - t_j) u_{j,i,j} - t_j^{-1} (t_j - 1)^2 u_{j,i,k} + t_j u_{k,i,j} + (t_j - 1) u_{k,i,k},
  \sqrt{u_{i,k,k} | u_{k,i,k}} => (t_j^{-1} - 1) u_{j,i,k} + u_{k,i,k},
  \sqrt{u_{j,i,l} | u_{i,j,l}} => u_{j,i,l},
  \sqrt{u_{k,i,l} | u_{i,k,l}} => (1 - t_j) u_{j,i,l} + t_j u_{k,i,l},
  \sqrt{u_{i1,i2,j}} => u_{i2,i1,j} + (1 - t_j^{-1}) u_{i2,i1,k},
  \sqrt{u_{i1,i2,k}} => t_j^{-1} u_{i2,i1,k},
  \sqrt{u_{i1,i2,l}} => u_{i2,i1,l},
  u_{_,_,_} => Echo@"Error"};
```

```
Unset[TG_{j,k}];
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SetDelayed @@
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```
{TG_{j,k}, Expand@*ReplaceAll[u_{jj,ii,kk} /; ! OrderedQ[{jj, ii}] => u_{ii,jj,kk}]@*
  ReplaceAll[TGRule]};
```

V_i with $V = \text{Gassner}$, matches w/ $S^2 V \oplus V^*$.

$$V_i: V_j \mapsto V_j$$

$$V_k \mapsto (1 - t_j) V_j + t_j V_k$$

$$V^*: V_j \mapsto V_j + (1 - t_j^{-1}) V_K$$

$$V_K \mapsto t_j^{-1} V_K$$