

VS = True;

Jacobi@@{a[f, 1, 1], a[g, 1, 2], a[h, 2, 1]}

0

Jacobi@@{a[f, 1, 2], a[g, 1, 3], a[h, 2, 3]}

Tails commute on $\delta_{aa}[fgh, 1, 3, 1, 2]$

Commute head/tail on $\delta_{aa}[fgh, 2, 3, 1, 2]$

Commute heads on $\delta_{aa}[-fgh, 2, 3, 1, 3]$

Commute heads on $\delta_{aa}[fgh, 2, 3, 1, 3]$

0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 2, 1]}

1322 swinging on $\delta_{aa}[-2, 1, 3, 2, 2]$

Commute head/tail on $\delta_{aa}[-2, 2, 3, 1, 2]$

Commute heads on $\delta_{aa}[2, 2, 3, 1, 3]$

Tails commute on $\delta_{aa}[-1, 1, 3, 1, 2]$

1321 swinging on $\delta_{aa}[-1, 1, 3, 2, 1]$

2113 swinging on $\delta_{aa}[1, 2, 1, 1, 3]$

0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], a[1, 3, 1]} // S

1332 swinging on $\delta_{aa}[-2, 1, 3, 3, 2]$

Tails commute on $\delta_{aa}[-1, 1, 3, 1, 2]$

3112 swinging on $\delta_{aa}[-1, 3, 1, 1, 2]$

1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$

0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], ca[1, 1, 3, 1]}

1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$

1332 swinging on $\delta_{aa}[b_1, 1, 3, 3, 2]$

Tails commute on $\delta_{aa}[-b_3, 1, 3, 1, 2]$

1231 swinging on $\delta_{aa}[-b_1, 1, 2, 3, 1]$

1211 sliding on $\delta_{aa}[b_3, 1, 2, 1, 1]$

0

Jacobi@@{a[f[b₁, b₂, b₃], 1, 2], a[g[b₁, b₂, b₃], 1, 3], $\delta_{aa}[h[b_1, b_2, b_3], 3, 1, 3, 1]}$

Tails commute on $\delta_{aa}[-f[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1, 3, 2, 3, 1]$
 1231 swinging on $\delta_{aa}[f[b_1, b_2, b_3] h[b_1, b_2, b_3] b_3, 1, 2, 3, 1]$
 3112 swinging on $\delta_{aa}[f[b_1, b_2, b_3] h[b_1, b_2, b_3] b_3, 3, 1, 1, 2]$
 1332 swinging on $\delta_{aa}[4 f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 1, 3, 3, 2]$
 2221 sliding on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1^2, 3, 3, 3, 2]$
 Tails commute on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_3^2, 1, 3, 1, 2]$
 Tails commute on $\delta_{aa}[2 f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1^2, 3, 2, 3, 1]$
 1231 swinging on $\delta_{aa}[-2 f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 1, 2, 3, 1]$
 1332 swinging on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 1, 3, 3, 2]$
 Locality on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 3, 2, 1, 1]$
 Commute head/tail on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 3, 2, 1, 3]$
 Commute heads on $\delta_{aa}[f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 3, 2, 1, 2]$
 1332 swinging on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 1, 3, 3, 2]$
 Locality on $\delta_{aa}[-f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_1 b_3, 3, 3, 1, 2]$
 1211 sliding on $\delta_{aa}[f[b_1, b_2, b_3] g[b_1, b_2, b_3] h[b_1, b_2, b_3] b_3^2, 1, 2, 1, 1]$
 0

Jacobi@@{a[1, 1, 2], a[1, 2, 1], a[1, 3, 1]}

Locality on $\delta_{aa}[-2, 3, 1, 2, 2]$
 2231 swinging on $\delta_{aa}[-2, 2, 2, 3, 1]$
 3112 swinging on $\delta_{aa}[-1, 3, 1, 1, 2]$
 Commute heads on $\delta_{aa}[-1, 3, 1, 2, 1]$
 1231 swinging on $\delta_{aa}[1, 1, 2, 3, 1]$
 0

Jacobi@@{a[1, 1, 2], a[1, 1, 3], $\delta_{aa}[1, 2, 1, 3, 1]$ }

2231 swinging on $\delta_{aa}[-b_1, 2, 2, 3, 1]$
 1231 swinging on $\delta_{aa}[b_2, 1, 2, 3, 1]$
 Backie swinging on $\delta_{aa}[b_3, 2, 1, 1, 2]$
 Locality on $\delta_{aa}[b_3, 2, 2, 1, 1]$
 Backie swinging on $\delta_{aa}[-b_1^2, 2, 3, 3, 2]$
 1332 swinging on $\delta_{aa}[b_1 b_2, 1, 3, 3, 2]$
 Backie swinging on $\delta_{aa}[b_1 b_3, 2, 1, 1, 2]$
 Locality on $\delta_{aa}[b_1 b_3, 2, 2, 1, 1]$
 Tails commute on $\delta_{aa}[-b_2 b_3, 1, 3, 1, 2]$
 2231 swinging on $\delta_{aa}[b_1^2, 2, 2, 3, 1]$
 1231 swinging on $\delta_{aa}[-b_1 b_2, 1, 2, 3, 1]$
 Locality on $\delta_{aa}[-b_1 b_3, 2, 2, 1, 1]$
 Locality on $\delta_{aa}[-b_1 b_3, 2, 2, 1, 3]$
 1322 swinging on $\delta_{aa}[-b_1 b_3, 1, 3, 2, 2]$
 Commute head/tail on $\delta_{aa}[-b_1 b_3, 2, 3, 1, 2]$
 Commute heads on $\delta_{aa}[b_1 b_3, 2, 3, 1, 3]$
 Commute head/tail on $\delta_{aa}[-b_1 b_3, 2, 3, 1, 2]$
 Commute heads on $\delta_{aa}[b_1 b_3, 2, 3, 1, 3]$
 1322 swinging on $\delta_{aa}[2 b_1 b_3, 1, 3, 2, 2]$
 Commute head/tail on $\delta_{aa}[2 b_1 b_3, 2, 3, 1, 2]$
 Commute heads on $\delta_{aa}[-2 b_1 b_3, 2, 3, 1, 3]$
 1211 sliding on $\delta_{aa}[b_2 b_3, 1, 2, 1, 1]$
 0

Jacobi @@ {a[1, 1, 2], a[1, 2, 1], a[1, 3, 2]}

1231 swinging on $\delta_{aa}[-2, 1, 2, 3, 1]$
 Locality on $\delta_{aa}[2, 3, 2, 1, 1]$
 Commute heads on $\delta_{aa}[1, 3, 2, 1, 2]$
 Commute head/tail on $\delta_{aa}[1, 3, 2, 2, 1]$
 Commute heads on $\delta_{aa}[-1, 3, 1, 2, 1]$
 0

Jacobi @@ {a[1, 1, 2], a[1, 2, 1], ca[1, 1, 1, 1]}

1211 sliding on $\delta_{aa}[1, 1, 2, 1, 1]$

2111 sliding on $\delta_{aa}[-b_1, 2, 1, 1, 1]$

1211 sliding on $\delta_{aa}[-b_2, 1, 2, 1, 1]$

2111 sliding on $\delta_{aa}[2b_1, 2, 1, 1, 1]$

Locality on $\delta_{aa}[b_1, 2, 2, 1, 1]$

0

Jacobi @@ {**a**[1, 1, 2], **a**[1, 2, 3], **δ_{aa}** [1, 2, 1, 3, 1]}

2231 swinging on $\delta_{aa}[-b_1, 2, 2, 3, 1]$

1231 swinging on $\delta_{aa}[b_2, 1, 2, 3, 1]$

Backie swinging on $\delta_{aa}[b_3, 2, 1, 1, 2]$

Locality on $\delta_{aa}[b_3, 2, 2, 1, 1]$

2331 swinging on $\delta_{aa}[b_1 b_2, 2, 3, 3, 1]$

Backie swinging on $\delta_{aa}[-b_2^2, 1, 3, 3, 1]$

2111 sliding on $\delta_{aa}[-b_2 b_3, 2, 1, 1, 1]$

2113 swinging on $\delta_{aa}[-b_2 b_3, 2, 1, 1, 3]$

2231 swinging on $\delta_{aa}[b_1 b_2, 2, 2, 3, 1]$

1231 swinging on $\delta_{aa}[-b_2^2, 1, 2, 3, 1]$

Tails commute on $\delta_{aa}[-2b_1 b_3, 2, 3, 2, 1]$

2221 sliding on $\delta_{aa}[-b_1 b_3, 2, 2, 2, 1]$

Backie swinging on $\delta_{aa}[b_2 b_3, 1, 2, 2, 1]$

2111 sliding on $\delta_{aa}[b_2 b_3, 2, 1, 1, 1]$

Locality on $\delta_{aa}[2b_2 b_3, 2, 3, 1, 1]$

0

Jacobi @@ {**a**[1, 1, 2], **a**[1, 2, 3], **δ_{aa}** [1, 3, 2, 3, 2]}

```

Commute heads on  $\delta_{aa}[-b_3, 3, 2, 1, 2]$ 
2221 sliding on  $\delta_{aa}[-b_1 b_2, 3, 3, 3, 2]$ 
Backie swinging on  $\delta_{aa}[b_1 b_3, 2, 3, 3, 2]$ 
2111 sliding on  $\delta_{aa}[b_1 b_3, 3, 2, 2, 2]$ 
Backie swinging on  $\delta_{aa}[b_1 b_3, 3, 2, 2, 3]$ 
Locality on  $\delta_{aa}[b_1 b_3, 3, 3, 2, 2]$ 
Commute heads on  $\delta_{aa}[-b_2 b_3, 3, 2, 1, 2]$ 
2221 sliding on  $\delta_{aa}[b_1 b_2, 3, 3, 3, 2]$ 
2111 sliding on  $\delta_{aa}[-b_1 b_3, 3, 2, 2, 2]$ 
1332 swinging on  $\delta_{aa}[-b_2 b_3, 1, 3, 3, 2]$ 
Backie swinging on  $\delta_{aa}[-2 b_1 b_3, 3, 2, 2, 3]$ 
Locality on  $\delta_{aa}[-2 b_1 b_3, 3, 3, 2, 2]$ 
Commute head/tail on  $\delta_{aa}[b_2 b_3, 3, 2, 1, 3]$ 
Commute heads on  $\delta_{aa}[-b_2 b_3, 3, 2, 1, 2]$ 
1332 swinging on  $\delta_{aa}[b_2 b_3, 1, 3, 3, 2]$ 
Locality on  $\delta_{aa}[b_2 b_3, 3, 3, 1, 2]$ 
Commute heads on  $\delta_{aa}[2 b_2 b_3, 3, 2, 1, 2]$ 
2212 sliding on  $\delta_{aa}[-b_3^2, 2, 2, 1, 2]$ 
Commute head/tail on  $\delta_{aa}[-b_3^2, 2, 3, 1, 2]$ 
Commute heads on  $\delta_{aa}[b_3^2, 2, 3, 1, 3]$ 
0

VS = False;
JacErrors = DeleteCases [
  bas1 = FormalBasis[4, f];
  bas2 = FormalBasis[4, g];
  bas3 = FormalBasis[4, h];
  Flatten[
    Table[Jacobi[bas1[[i], bas2[[j], bas3[[k]],
      {i, Length[bas1] - 1}, {j, i + 1, Length@bas2}, {k, i + 1, Length@bas3}]
    ],
  0]
{}

(* was 0 for FormalBasis[3], 2156 for FormalBasis[4] *)
Length@JacErrors
0

```