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ca[0, __] := 0;
ca /: ca[f_, is_] + ca[g_, is_] := ca[f+g, is];
ca /: -ca[f_, is_] := ca[Expand[-f], is];
B[a[f_, j_, k_], c[l_]] /; DistinctQ[j, k, l] := 0;
B[a[f_, j_, k_], c[j_]] :=  $\gamma[-f, j, k]$ ;
B[a[f_, j_, k_], c[k_]] :=  $\gamma[f, j, k]$ ;
B[x_, ca[f_, i_, j_, k_]] := Plus[
  B[x, c[i]] /.  $\gamma[g_, l_, m_] \Rightarrow \gamma_a[f g, l, m, j, k]$ ,
  B[x, a[f, j, k]] /. {a[g_, l_, m_] => ca[g, i, l, m],  $-\gamma \rightarrow 0$ ,  $-\gamma_a \rightarrow 0$ }
];
 $\delta_{aa}[f_, i_, j_, k_, l_] /; OrderedQ[{l, j}] \wedge DistinctQ[i, j, k, l] :=$ 
   $\delta_{aa}[f, k, l, i, j]$ ;
 $\delta_{aa}[f_, i_, j_, i_, l_] /; OrderedQ[{l, j}] \wedge DistinctQ[j, l] := \delta_{aa}[f, i, l, i, j]$ ;
(* missing a 4T! *)
E $\gamma$  = {
   $\gamma[f_, j_, k_, l_] \Rightarrow ca[f, l, j, k] + ca[Expand[-f], k, j, l]$ ,
   $\gamma_a[f_, i_, j_, k_, l_] \Rightarrow \delta_{aa}[f, i, j, k, l] + ca[Expand[-b_i f], j, k, l]$ 
};
Check3[y_ $\gamma$ ] :=
  {t1 = B[a[f, j, k], y] /. E $\gamma$ , t2 = B[a[f, j, k], y /. E $\gamma$ ] /. E $\gamma$ , t1 == t2 // Simplify}
{x1, x2, x3} = {a[1, 1, 2], a[1, 2, 3], a[h[b1, b2, b3], 1, 2]}
{a[1, 1, 2], a[1, 2, 3], a[h[b1, b2, b3], 1, 2]}

B[x1, x2] + B[x2, x1]
0

B[x1, B[x2, x3]] + B[x2, B[x3, x1]] + B[x3, B[x1, x2]] // LSimp
 $\gamma[-b_1^2 h^{(0,0,1)}[b_1, b_2, b_3] + b_1 b_2 h^{(0,0,1)}[b_1, b_2, b_3] +$ 
   $b_1^2 h^{(0,1,0)}[b_1, b_2, b_3] - b_1 b_2 h^{(0,1,0)}[b_1, b_2, b_3, 1, 2, 3] +$ 
   $\gamma_a[b_1 h^{(0,0,1)}[b_1, b_2, b_3] - b_2 h^{(0,0,1)}[b_1, b_2, b_3] - b_1 h^{(0,1,0)}[b_1, b_2, b_3] +$ 
   $b_2 h^{(0,1,0)}[b_1, b_2, b_3, 1, 2, 1, 3]$ 

B[x1, B[x2, x3]] + B[x2, B[x3, x1]] + B[x3, B[x1, x2]] // ColumnForm
 $\gamma[-b_1^2 h^{(0,0,1)}[b_1, b_2, b_3] + b_1 b_2 h^{(0,0,1)}[b_1, b_2, b_3] + b_1^2 h^{(0,1,0)}[b_1, b_2, b_3] - b_1 b_2 h^{(0,1,0)}[b_1, b_2$ 
 $\gamma_a[b_1 h^{(0,0,1)}[b_1, b_2, b_3] - b_2 h^{(0,0,1)}[b_1, b_2, b_3] - b_1 h^{(0,1,0)}[b_1, b_2, b_3] + b_2 h^{(0,1,0)}[b_1, b_2, b_3],$ 

B[x2, x3]
a[-h[b1, b2, b3] b1, 2, 3] + a[h[b1, b2, b3] b2, 1, 3] +
   $\gamma[h[b_1, b_2, b_3], 1, 2, 3] + \gamma_a[-h^{(0,0,1)}[b_1, b_2, b_3] + h^{(0,1,0)}[b_1, b_2, b_3], 2, 3, 1, 2]$ 

B[x1,  $\gamma[h[b_1, b_2, b_3], 1, 2, 3]$ ]
 $\gamma_a[-h[b_1, b_2, b_3], 1, 2, 1, 3]$ 

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B[x3, x1]

$$\gamma a [h^{(0,1,0)} [b_1, b_2, b_3] - h^{(1,0,0)} [b_1, b_2, b_3], 1, 2, 1, 2]$$

B[x1, x2]

$$a [b_1, 2, 3] + a [-b_2, 1, 3] + \gamma [-1, 1, 2, 3]$$

B[x1, B[x2, x3]]

$$\begin{aligned} & a [-h [b_1, b_2, b_3] b_1^2, 2, 3] + a [h [b_1, b_2, b_3] b_1 b_2, 1, 3] + \\ & \gamma [h [b_1, b_2, b_3] b_1 + h [b_1, b_2, b_3] b_2 - b_1^2 h^{(0,0,1)} [b_1, b_2, b_3] + b_1^2 h^{(0,1,0)} [b_1, b_2, b_3], \\ & 1, 2, 3] + \gamma a [-b_1 h^{(0,0,1)} [b_1, b_2, b_3] + b_1 h^{(0,1,0)} [b_1, b_2, b_3], 2, 3, 1, 2] + \\ & \gamma a [-h [b_1, b_2, b_3] + b_1 h^{(0,1,0)} [b_1, b_2, b_3] - b_1 h^{(1,0,0)} [b_1, b_2, b_3], 1, 2, 2, 3] + \\ & \gamma a [-2 h [b_1, b_2, b_3] + b_1 h^{(0,0,1)} [b_1, b_2, b_3] - b_1 h^{(0,1,0)} [b_1, b_2, b_3] - \\ & b_2 h^{(0,1,0)} [b_1, b_2, b_3] + b_2 h^{(1,0,0)} [b_1, b_2, b_3], 1, 2, 1, 3] \end{aligned}$$

{y1, y2, y3} = {a[1, 1, 2], a[1, 2, 3], a[b3, 1, 2]};**B[y1, B[y2, y3]] + B[y2, B[y3, y1]] + B[y3, B[y1, y2]]**

$$\gamma [-b_1^2 + b_1 b_2, 1, 2, 3] + \gamma a [b_1 - b_2, 1, 2, 1, 3]$$

B[y2, y3]

$$a [-b_1 b_3, 2, 3] + a [b_2 b_3, 1, 3] + \gamma [b_3, 1, 2, 3] + \gamma a [-1, 2, 3, 1, 2]$$

B[y1, B[y2, y3]]

$$\begin{aligned} & a [-b_1^2 b_3, 2, 3] + a [b_1 b_2 b_3, 1, 3] + \gamma [-b_1^2 + b_1 b_3 + b_2 b_3, 1, 2, 3] + \\ & \gamma a [-b_1, 2, 3, 1, 2] + \gamma a [b_1 - 2 b_3, 1, 2, 1, 3] + \gamma a [-b_3, 1, 2, 2, 3] \end{aligned}$$