Kapovitch's class, Mon Feb 9: Priyam Patel on RF, Separability, LERF

February-09-15 11:10 AM

Defs ① A group G is residually finite (RF) if $\forall g \in G\setminus\{1\}$ $\exists G' \leq_{f.i.} G$ s.t. $g \notin G'$.

$\Longleftrightarrow$ (same with $\triangleleft$)

② $H \leq G$ is seperable if $\forall g \in G\setminus H$ $\exists G' \geq H$ w/ $g \notin G'$

③ G is LERF / subgroup seperable if every f.g. $H \leq G$ is seperable ($\Rightarrow$ RF)

Claim Finitely presented residually finite group has solvable word problem!

Topology RF: $\pi_1(M) = G$, $\gamma \in \pi_1(M)$ s.t. $\gamma \notin H$

$\Rightarrow$ $\exists$ finite cover s.t. γ lifts to an open curve.

Separability: Roughly, can lift self-intersecting surfaces to embedded surfaces

Lemma (Scott '78) Let X be a T_2 top. space w/ regular cover $\tilde{X}$ w/ group G. (Typically, $X = M$, $\tilde{X}$ univ. cover) Then

G is LERF iff given $S < G$ f.g. & compact $C \subset \tilde{X}/S$ the $\exists$ finite cover X_1 of X s.t. $\tilde{X}/S \longrightarrow X$ factors through X_1 & C projects by a homeomorphism to X_1

$$\begin{tikzcd} & \tilde{X}/S \arrow[rr] \arrow[dr] & & X \\ C \arrow[rr, "\text{homeo}"'] & & X_1 \arrow[ur] & \end{tikzcd}$$

Pf ...