

Pensieve header: Infinitesimal Elliptic computations, version Lie.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2015-02\\InfinitesimalElliptic"];
<< FreeLieBasic.m

AllLyndonWords[n_, d_Integer] := AllLyndonWords[d, Union@@Table[{x_i, y_i}, {i, n}]]

AllRelators2[n_] := Flatten[{
  Table[{cx_{i,j}, cy_{i,j}, cxy_{i,j}}, {i, 1, n-1}, {j, i+1, n}],
  Table[c_{n,i}, {i, n}]
}];

AllRelators3[n_] := Flatten[Table[
  {ctx_{i,j,k}, cty_{i,j,k}, ctx_{j,k,i}, cty_{j,k,i}, ctx_{k,i,j}, cty_{k,i,j}},
  {i, 1, n-2}, {j, i+1, n-1}, {k, j+1, n}
]];

ad[AW[], x_] := x;
ad[AW[lft___, g_], x_] := ad[AW[lft], b[LW[g], x]];

AllWords[n_, d_] := AW@@@Tuples[Union@@Table[{x_i, y_i}, {i, n}], d];
AllRelators2[n_, d_] /; d < 2 := {};
AllRelators2[n_, d_] :=
  Flatten[Outer[ad, AllWords[n, d-2], LW/@AllRelators2[n]]];
AllRelators3[n_, d_] /; d < 3 := {};
AllRelators3[n_, d_] :=
  Flatten[Outer[ad, AllWords[n, d-3], LW/@AllRelators3[n]]];
AllRelators[n_, d_] := Join[AllRelators2[n, d], AllRelators3[n, d]];

LieMorphism[diff, {
  LW@cx_{i,j} -> b[LW@x_i, LW@x_j], LW@cy_{i,j} -> b[LW@y_i, LW@y_j],
  LW@cxy_{i,j} -> b[LW@x_i, LW@y_j] - b[LW@x_j, LW@y_i],
  LW@ctx_{i,j,k} -> b[b[LW@x_i, LW@y_j], LW@x_k],
  LW@cty_{i,j,k} -> b[b[LW@x_i, LW@y_j], LW@y_k],
  LW@c_{n,i} -> \sum_{j=1}^n b[LW@x_i, LW@y_j]
}];

```

```

Dim[n_, d_] := Dim[n, d] = Module[{bas, tdim, im, v, vc},
  tdim = Length[bas = AllLyndonWords[n, d]];
  im = (diff /@ AllRelators[n, d]) /. Thread[bas -> Table[v[i], {i, tdim}]];
  If[im == {},
    tdim,
    tdim - MatrixRank[
      SparseArray[
        Flatten[Table[
          (*Cases[{im[[i]]}, c_.*v[j_] -> {{i, j} -> c}, ∞],
          Wrong! c_.*v[j_] matches some terms twice,
          producing extra matrix entries. *)
          vc = im[[i]];
          ({i, #[[1]]} -> Coefficient[vc, #]) & /@ Cases[{vc}, _v, ∞],
          {i, Length[im]}]]],
        {Length[im], tdim}]
      ]
    ]
  ]

```

```

PrimitivesToFull[p_List] := Module[
  {lp, h, ser},
  lp = Length[p];
  ser = Normal[Series[ $\prod_{k=1}^{lp} \left(\frac{1}{1-h^k}\right)^{p^{[k]}}$ , {h, 0, lp}]];
  Table[Coefficient[ser, h, i], {i, 0, lp}]
];

PrimitivesToFull[Table[Dim[2, d], {d, 7}]] // Timing
{56.768764, {1, 4, 11, 26, 57, 120, 247, 502}}

PrimitivesToFull[Table[Dim[3, d], {d, 6}]] // Timing
{573.522076, {1, 6, 24, 80, 241, 682, 1852}}

3^Range[10]
{3, 9, 27, 81, 243, 729, 2187, 6561, 19683, 59049}

PrimitivesToFull[Table[Dim[4, d], {d, 4}]] // Timing
{2.667617, {1, 8, 42, 180, 685}}

```

Testing

```

AllLyndonWords[5, {1, 2}] // TopBracketForm
{111112, 111222, 11212, 112222, 121222, 122222}

```

AllLyndonWords [2, 3] // Length

20

ad[AW[1, 1, 1, 1, 1, 1], LW[2]]

$\overline{\overline{\overline{\overline{\overline{\overline{11111112}}}}}}}$

ad[AW[2, 2, 2, 2, 2], LW[1]]

$\overline{\overline{\overline{\overline{\overline{-1222222}}}}}}}$

ad[AW[2, 2, 1, 2, 1, 2], LW[1]]

$\overline{\overline{\overline{\overline{\overline{11122222}}}}}} + \overline{\overline{\overline{\overline{\overline{211212222}}}}}} - \overline{\overline{\overline{\overline{\overline{11221222}}}}}} - \overline{\overline{\overline{\overline{\overline{211222212}}}}}} + \overline{\overline{\overline{\overline{\overline{212121222}}}}}}$

{AllRelators2[3, 3], AllRelators3[3, 3]}

$\{ \{ \overline{x_1 c x_{1,2}}, \overline{x_1 c y_{1,2}}, \overline{x_1 c x y_{1,2}}, \overline{x_1 c x_{1,3}}, \overline{x_1 c y_{1,3}}, \overline{x_1 c x y_{1,3}}, \overline{x_1 c x_{2,3}}, \overline{x_1 c y_{2,3}}, \overline{x_1 c x y_{2,3}},$
 $\overline{x_1 c_{3,1}}, \overline{x_1 c_{3,2}}, \overline{x_1 c_{3,3}}, \overline{x_2 c x_{1,2}}, \overline{x_2 c y_{1,2}}, \overline{x_2 c x y_{1,2}}, \overline{x_2 c x_{1,3}}, \overline{x_2 c y_{1,3}}, \overline{x_2 c x y_{1,3}},$
 $\overline{x_2 c x_{2,3}}, \overline{x_2 c y_{2,3}}, \overline{x_2 c x y_{2,3}}, \overline{x_2 c_{3,1}}, \overline{x_2 c_{3,2}}, \overline{x_2 c_{3,3}}, \overline{x_3 c x_{1,2}}, \overline{x_3 c y_{1,2}}, \overline{x_3 c x y_{1,2}},$
 $\overline{x_3 c x_{1,3}}, \overline{x_3 c y_{1,3}}, \overline{x_3 c x y_{1,3}}, \overline{x_3 c x_{2,3}}, \overline{x_3 c y_{2,3}}, \overline{x_3 c x y_{2,3}}, \overline{x_3 c_{3,1}}, \overline{x_3 c_{3,2}}, \overline{x_3 c_{3,3}},$
 $\overline{y_1 c x_{1,2}}, \overline{y_1 c y_{1,2}}, \overline{y_1 c x y_{1,2}}, \overline{y_1 c x_{1,3}}, \overline{y_1 c y_{1,3}}, \overline{y_1 c x y_{1,3}}, \overline{y_1 c x_{2,3}}, \overline{y_1 c y_{2,3}}, \overline{y_1 c x y_{2,3}},$
 $\overline{y_1 c_{3,1}}, \overline{y_1 c_{3,2}}, \overline{y_1 c_{3,3}}, \overline{y_2 c x_{1,2}}, \overline{y_2 c y_{1,2}}, \overline{y_2 c x y_{1,2}}, \overline{y_2 c x_{1,3}}, \overline{y_2 c y_{1,3}}, \overline{y_2 c x y_{1,3}},$
 $\overline{y_2 c x_{2,3}}, \overline{y_2 c y_{2,3}}, \overline{y_2 c x y_{2,3}}, \overline{y_2 c_{3,1}}, \overline{y_2 c_{3,2}}, \overline{y_2 c_{3,3}}, \overline{y_3 c x_{1,2}}, \overline{y_3 c y_{1,2}}, \overline{y_3 c x y_{1,2}},$
 $\overline{y_3 c x_{1,3}}, \overline{y_3 c y_{1,3}}, \overline{y_3 c x y_{1,3}}, \overline{y_3 c x_{2,3}}, \overline{y_3 c y_{2,3}}, \overline{y_3 c x y_{2,3}}, \overline{y_3 c_{3,1}}, \overline{y_3 c_{3,2}}, \overline{y_3 c_{3,3}} \},$
 $\{ \overline{c t x_{1,2,3}}, \overline{c t y_{1,2,3}}, \overline{c t x_{2,3,1}}, \overline{c t y_{2,3,1}}, \overline{c t x_{3,1,2}}, \overline{c t y_{3,1,2}} \}$

diff[AllRelators[3, 3]]

{ $\overline{x_1 x_1 x_2}$, $\overline{x_1 y_1 y_2}$, $\overline{x_1 x_1 y_2} - \overline{x_1 x_2 y_1}$, $\overline{x_1 x_1 x_3}$, $\overline{x_1 y_1 y_3}$, $\overline{x_1 x_1 y_3} - \overline{x_1 x_3 y_1}$, $\overline{x_1 x_2 x_3}$,
 $\overline{x_1 y_2 y_3}$, $\overline{x_1 x_2 y_3} - \overline{x_1 x_3 y_2}$, $\overline{x_1 x_1 y_1} + \overline{x_1 x_1 y_2} + \overline{x_1 x_1 y_3}$, $\overline{x_1 x_2 y_1} + \overline{x_1 x_2 y_2} + \overline{x_1 x_2 y_3}$,
 $\overline{x_1 x_3 y_1} + \overline{x_1 x_3 y_2} + \overline{x_1 x_3 y_3}$, $-\overline{x_1 x_2 x_2}$, $\overline{x_2 y_1 y_2}$, $-\overline{x_1 y_2 x_2} - \overline{x_2 x_2 y_1}$, $-\overline{x_1 x_3 x_2}$, $\overline{x_2 y_1 y_3}$,
 $-\overline{x_1 y_3 x_2} - \overline{x_2 x_3 y_1}$, $\overline{x_2 x_2 x_3}$, $\overline{x_2 y_2 y_3}$, $\overline{x_2 x_2 y_3} - \overline{x_2 x_3 y_2}$, $-\overline{x_1 y_1 x_2} - \overline{x_1 y_2 x_2} - \overline{x_1 y_3 x_2}$,
 $\overline{x_2 x_2 y_1} + \overline{x_2 x_2 y_2} + \overline{x_2 x_2 y_3}$, $\overline{x_2 x_3 y_1} + \overline{x_2 x_3 y_2} + \overline{x_2 x_3 y_3}$, $-\overline{x_1 x_2 x_3} - \overline{x_1 x_3 x_2}$, $\overline{x_3 y_1 y_2}$,
 $-\overline{x_1 y_2 x_3} + \overline{x_2 y_1 x_3}$, $-\overline{x_1 x_3 x_3}$, $\overline{x_3 y_1 y_3}$, $-\overline{x_1 y_3 x_3} - \overline{x_3 x_3 y_1}$, $-\overline{x_2 x_3 x_3}$, $\overline{x_3 y_2 y_3}$,
 $-\overline{x_2 y_3 x_3} - \overline{x_3 x_3 y_2}$, $-\overline{x_1 y_1 x_3} - \overline{x_1 y_2 x_3} - \overline{x_1 y_3 x_3}$, $-\overline{x_2 y_1 x_3} - \overline{x_2 y_2 x_3} - \overline{x_2 y_3 x_3}$,
 $\overline{x_3 x_3 y_1} + \overline{x_3 x_3 y_2} + \overline{x_3 x_3 y_3}$, $-\overline{x_1 x_2 y_1} - \overline{x_1 y_1 x_2}$, $\overline{y_1 y_1 y_2}$, $-\overline{x_1 y_2 y_1} + \overline{x_2 y_1 y_1}$,
 $-\overline{x_1 x_3 y_1} - \overline{x_1 y_1 x_3}$, $\overline{y_1 y_1 y_3}$, $-\overline{x_1 y_3 y_1} + \overline{x_3 y_1 y_1}$, $-\overline{x_2 x_3 y_1} - \overline{x_2 y_1 x_3}$, $\overline{y_1 y_2 y_3}$,
 $-\overline{x_2 y_3 y_1} + \overline{x_3 y_2 y_1}$, $-\overline{x_1 y_1 y_1} - \overline{x_1 y_2 y_1} - \overline{x_1 y_3 y_1}$, $-\overline{x_2 y_1 y_1} - \overline{x_2 y_2 y_1} - \overline{x_2 y_3 y_1}$,
 $-\overline{x_3 y_1 y_1} - \overline{x_3 y_2 y_1} - \overline{x_3 y_3 y_1}$, $-\overline{x_1 x_2 y_2} - \overline{x_1 y_2 x_2}$, $-\overline{y_1 y_2 y_2}$, $-\overline{x_1 y_2 y_2} + \overline{x_2 y_1 y_2} + \overline{x_2 y_2 y_1}$,
 $-\overline{x_1 x_3 y_2} - \overline{x_1 y_2 x_3}$, $-\overline{y_1 y_3 y_2}$, $-\overline{x_1 y_3 y_2} + \overline{x_3 y_1 y_2} + \overline{x_3 y_2 y_1}$, $-\overline{x_2 x_3 y_2} - \overline{x_2 y_2 x_3}$,
 $\overline{y_2 y_2 y_3}$, $-\overline{x_2 y_3 y_2} + \overline{x_3 y_2 y_2}$, $-\overline{x_1 y_1 y_2} - \overline{x_1 y_2 y_1} - \overline{x_1 y_2 y_2} - \overline{x_1 y_3 y_2}$,
 $-\overline{x_2 y_1 y_2} - \overline{x_2 y_2 y_1} - \overline{x_2 y_2 y_2} - \overline{x_2 y_3 y_2}$, $-\overline{x_3 y_1 y_2} - \overline{x_3 y_2 y_1} - \overline{x_3 y_2 y_2} - \overline{x_3 y_3 y_2}$,
 $-\overline{x_1 x_2 y_3} - \overline{x_1 y_3 x_2}$, $-\overline{y_1 y_2 y_3} - \overline{y_1 y_3 y_2}$, $-\overline{x_1 y_2 y_3} - \overline{x_1 y_3 y_2} + \overline{x_2 y_1 y_3} + \overline{x_2 y_3 y_1}$,
 $-\overline{x_1 x_3 y_3} - \overline{x_1 y_3 x_3}$, $-\overline{y_1 y_3 y_3}$, $-\overline{x_1 y_3 y_3} + \overline{x_3 y_1 y_3} + \overline{x_3 y_3 y_1}$, $-\overline{x_2 x_3 y_3} - \overline{x_2 y_3 x_3}$,
 $-\overline{y_2 y_3 y_3}$, $-\overline{x_2 y_3 y_3} + \overline{x_3 y_2 y_3} + \overline{x_3 y_3 y_2}$, $-\overline{x_1 y_1 y_3} - \overline{x_1 y_2 y_3} - \overline{x_1 y_3 y_1} - \overline{x_1 y_3 y_2} - \overline{x_1 y_3 y_3}$,
 $-\overline{x_2 y_1 y_3} - \overline{x_2 y_2 y_3} - \overline{x_2 y_3 y_1} - \overline{x_2 y_3 y_2} - \overline{x_2 y_3 y_3}$,
 $-\overline{x_3 y_1 y_3} - \overline{x_3 y_2 y_3} - \overline{x_3 y_3 y_1} - \overline{x_3 y_3 y_2} - \overline{x_3 y_3 y_3}$, $\overline{x_1 y_2 x_3}$,
 $\overline{x_1 y_2 y_3} + \overline{x_1 y_3 y_2}$, $-\overline{x_1 x_2 y_3}$, $\overline{x_2 y_3 y_1}$, $-\overline{x_2 x_3 y_1}$, $\overline{x_3 y_1 y_2} + \overline{x_3 y_2 y_1}$ }

Dim[3, 3]

16

Dim[2, 1]

4

Dim[2, 2]

1

PrimitivesToFull[Table[Dim[1, d], {d, 6}]]

{1, 2, 3, 4, 5, 6, 7}

Table[Dim[1, d], {d, 6}]

{2, 0, 0, 0, 0, 0}

dd = 7; Table[Dim[2, d], {d, dd}] - Table[Dim[1, d], {d, dd}]

{2, 1, 2, 3, 6, 9, 18}

dd = 6; Table[Dim[3, d], {d, dd}] - Table[Dim[2, d], {d, dd}]

{2, 2, 4, 7, 16, 30}

PrimitivesToFull@{2, 2, 4, 7, 16, 30}

{1, 2, 5, 12, 29, 70, 169}