Upper and lower

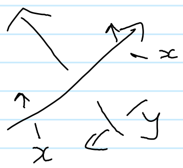
July-08-14 7:37 AM

Buran: $\begin{pmatrix} 1-t & 1 \\ t & 0 \end{pmatrix}$ Inverse conjugate transpose: $\begin{pmatrix} 0 & 1 \\ t & 1-t \end{pmatrix}$

Must really be "upper Buran" and "lower Buran".

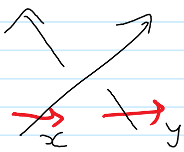
Buran for W:

Upper Gassner:



$$x \longmapsto x \longrightarrow \begin{pmatrix} 1 & 0 \end{pmatrix}$$
$$y \longmapsto xyx^{-1} \longrightarrow \begin{pmatrix} 1-t_y & t_x \end{pmatrix}$$

Lower Gassner:



$$x \longmapsto y^{-1}xy \longrightarrow \begin{pmatrix} t_y^{-1} & -t_y^{-1}+t_y^{-1}t_x \end{pmatrix}$$
$$y \longmapsto y \longrightarrow \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 1-t_y & t_x \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ t_x^{-1}(t_y-1) & t_x^{-1} \end{pmatrix}$$

From TestingFoxGassner.nb:

```
In[1]:= U[i_, j_] := ReplacePart[IdentityMatrix[3], {
          {i, i} → 1, {i, j} → 0,
          {j, i} → 1 - t_j, {j, j} → t_i
        }]

In[5]:= U[1, 2].U[1, 3].U[2, 3] // Simplify // MatrixForm
```

Out[5]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 1-t_2 & t_1 & 0 \\ 1-t_3 & -t_1(-1+t_3) & t_1 t_2 \end{pmatrix}$$

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In[4]:= U[2, 3].U[1, 3].U[1, 2] // Simplify // MatrixForm
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Out[4]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 1-t_2 & t_1 & 0 \\ 1-t_3 & -t_1(-1+t_3) & t_1 t_2 \end{pmatrix}$$

I don't understand how/if the Fox calculus arises within w-calculus!

via $z^{-1}Ez$?

$$v^{-1}u^{-1}E(uv)$$

$$e^{-x}Ee^{x} = \qquad = v^{-1}u^{-1}(Eu)v + v^{-1}u^{-1}uEv$$

No. $= (\bar{u})^{v} + \bar{v}$

via normalized differentiation? $w \mapsto w^{-1}\delta w$

$$e^{x}e^{y} \mapsto e^{x+\delta x}e^{y+\delta y}e^{-y}e^{-x} \ldots$$

Yes.

Question Is there (nice) homomorphism

$$\{\text{basis conjugating } \varphi: F_n \to F_n\} \to M_{n\times n}[\mathbb{Z}\mathbb{Z}^n]$$

Ans Let $\pi: F_n \to \mathbb{Z}^n$ be the gen-count. Let $K = \ker \pi = [F_n, F_n]$. Then

$$K/[K,K] =$$

BCH/β: $\quad F_n \longrightarrow (\mathbb{Z}\mathbb{Z}_n)^n$

Question Is there a good composition formula ("chain rule") for Fox derivatives?