



$$Z_M(t) = \exp\left(\sum_{n \geq 1} \text{tr}(M^n) \frac{t^n}{n}\right)$$

Thm If  $G$  is a free group, then for any  $M \in M_{d \times d}(\mathbb{Z}G)$ ,  $Z_M(t)$  is an algebraic function of  $t$  & it is a power series w/ integer coefficients.

Example  $M = (X + X^{-1} + Y + Y^{-1}) \in M_{1 \times 1}(\mathbb{Z}(G))$

$$Z_M(t) = \frac{2}{3} \frac{1 + 2\sqrt{1 - 12t^2}}{1 - 6t^2 + \sqrt{1 - 12t^2}}$$


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Idea of proof: