Unitarity of Burau

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$$U_i = \begin{pmatrix} I_i & & & \\ & 1-t & t & \\ & 1 & 0 & \\ & & & I_{n-i-1} \end{pmatrix}, \quad U_i^{-1} = \begin{pmatrix} I_i & & & \\ & 0 & 1 & \\ & \bar t & 1-\bar t & \\ & & & I_{n-i-1} \end{pmatrix}.$$

Unitarity: $\bar U \Omega_n U^T = \Omega_n$ with $\Omega_n = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1-t & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1-t & 1-t & \cdots & 1 \end{pmatrix}$

$$U \Omega_n^T \bar U^T = \Omega_n^T \qquad \bar U^{-1} \Omega_n = \Omega_n U^T$$

$\bar U^{-1} \Omega_n$:

$$\begin{pmatrix} 1 & 0 & & 0 \\ 1-t & 1 & & 0 \\ \vdots & \vdots & & \vdots \\ 1-t & 1-t & & 1 \end{pmatrix}$$

$t + (1-t)^2 =$

$$\begin{pmatrix} 0 & 1 \\ \bar t & 1-\bar t \end{pmatrix}$$

$$\begin{pmatrix} (1-t) & t \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} I_{i-1} & & & \\ & 0 & 1 & \\ & t & 1-t & \\ & & & I_{n-i-1} \end{pmatrix} \begin{pmatrix} (\Omega_n)_{1,j;\, i-1,*} & & & \\ 1-t \ldots & 1-t & 1 & 0\ 0 \\ 1-t \ldots 1-t & 1-t+t^2 & 1-t & 0 \\ (\Omega_n)_{i+1,j;\, n,*} & & & \end{pmatrix} \begin{array}{l} \\ i \\ i+1 \\ \\ \end{array}$$

$\Omega_n U^T$:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-t & 1 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \Omega_i & 0 & 0 & 0 \\ 1-t & 1 & 0 & 0 \\ 1-t & 1-t & 1 & 0 \\ 1-t & 1-t & 1-t & \Omega_{n-i-1} \end{pmatrix} \begin{pmatrix} \Omega_i & 0 & 0 & 0 \\ 1-t & 1-t & 1 & 0 \\ 1-t & 1-t+t^2 & 1-t & 0 \\ 1-t & 1-t & 1-t & \Omega_{n-i-1} \end{pmatrix}$$

$$1-t \quad 1-t \quad 1-t \quad \lambda_{n-i+1} \Big| \Big| \quad 1-t \quad 1-t \quad 1-t \quad \lambda_{n-i+1} \Big|$$

checks with no understanding.