

Pensieve Header: The underslide planarity condition.

```
dir = SetDirectory["C:/drorbn/AcademicPensieve/2014-06/"];
```

```
<< MetaCalculi/MetaCalculi-Program.m
```

```
Format[ $\alpha_{a,b}$ , StandardForm] := Interpretation[ $\alpha_{10 a+b}$ ,  $\alpha_{ab}$ ];
```

```
{n = 4;  $\gamma_0 = \Gamma\left[\omega, \sum_{a=0}^n h_a \sigma_a, \sum_{a=1}^n \sum_{b=1}^n t_a h_b \alpha_{ab}\right], \gamma_1 = \gamma_0 // \text{ds}[2] // \text{ds}[4],$ 
```

```
U = Xm[1, u1] Xm[2, u2] Xm[3, u3] Xm[4, u4] //  $\Gamma // \text{dm}[u1, u2, u] // \text{dm}[u, u3, u] //$   
dm[u, u4, u],
```

```
t1 = U** ( $\gamma_0 * \Gamma[\epsilon[u]]$ ),
```

```
t2 = ( $\gamma_0 * \Gamma[\epsilon[u]]$ ) ** U,
```

```
ucond = FullSimplify[t1 == t2]} // ColumnForm
```

$$\begin{pmatrix} \omega & S_1 & S_2 & S_3 & S_4 \\ S_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ S_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ S_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ S_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{\omega (\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44})}{\sigma_2 \sigma_4} & & & & & & \\ S_1 & \frac{\alpha_{14} \alpha_{22} \alpha_{41} - \alpha_{12} \alpha_{24} \alpha_{41} - \alpha_{14} \alpha_{21} \alpha_{42} + \alpha_{11} \alpha_{24} \alpha_{42} + \alpha_{12} \alpha_{21} \alpha_{44} - \alpha_{14} \alpha_{11} \alpha_{22} \alpha_{44}}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} & & & & & \\ S_2 & & \frac{\alpha_{24} \alpha_{41} - \alpha_{21} \alpha_{44}}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} & & & & \\ S_3 & & & \frac{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}}{\alpha_{24} \alpha_{42} - \alpha_{22} \alpha_{44}} & & & \\ S_4 & & & & \frac{\alpha_{22} \alpha_{41} - \alpha_{21} \alpha_{42}}{-\alpha_{24} \alpha_{42} + \alpha_{22} \alpha_{44}} & & \\ \Sigma & & & & & \sigma_1 & \end{pmatrix}$$

$$\begin{pmatrix} 1 & S_1 & S_2 & S_3 & S_4 & S_u \\ S_1 & 1 & 0 & 0 & 0 & \frac{-1+T_1}{T_1} \\ S_2 & 0 & 1 & 0 & 0 & \frac{-1+T_2}{T_1 T_2} \\ S_3 & 0 & 0 & 1 & 0 & \frac{-1+T_3}{T_1 T_2 T_3} \\ S_4 & 0 & 0 & 0 & 1 & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ S_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & 1 & 1 & 1 & 1 & \frac{1}{T_1 T_2 T_3 T_4} \end{pmatrix}$$

$$\begin{pmatrix} \omega & S_1 & S_2 & S_3 & S_4 & S_u \\ S_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-T_2 T_3 T_4 \alpha_{11} + T_1 T_2 T_3 T_4 \alpha_{11} - T_3 T_4 \alpha_{12} + T_2 T_3 T_4 \alpha_{12} - T_4 \alpha_{13} + T_3 T_4 \alpha_{13} - \alpha_{14} + T_4 \alpha_{14}}{T_1 T_2 T_3 T_4} \\ S_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-T_2 T_3 T_4 \alpha_{21} + T_1 T_2 T_3 T_4 \alpha_{21} - T_3 T_4 \alpha_{22} + T_2 T_3 T_4 \alpha_{22} - T_4 \alpha_{23} + T_3 T_4 \alpha_{23} - \alpha_{24} + T_4 \alpha_{24}}{T_1 T_2 T_3 T_4} \\ S_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-T_2 T_3 T_4 \alpha_{31} + T_1 T_2 T_3 T_4 \alpha_{31} - T_3 T_4 \alpha_{32} + T_2 T_3 T_4 \alpha_{32} - T_4 \alpha_{33} + T_3 T_4 \alpha_{33} - \alpha_{34} + T_4 \alpha_{34}}{T_1 T_2 T_3 T_4} \\ S_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-T_2 T_3 T_4 \alpha_{41} + T_1 T_2 T_3 T_4 \alpha_{41} - T_3 T_4 \alpha_{42} + T_2 T_3 T_4 \alpha_{42} - T_4 \alpha_{43} + T_3 T_4 \alpha_{43} - \alpha_{44} + T_4 \alpha_{44}}{T_1 T_2 T_3 T_4} \\ S_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{pmatrix}$$

$$\begin{pmatrix} \omega & S_1 & S_2 & S_3 & S_4 & S_u \\ S_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-1+T_1}{T_1} \\ S_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-1+T_2}{T_1 T_2} \\ S_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-1+T_3}{T_1 T_2 T_3} \\ S_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ S_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{pmatrix}$$

$$\frac{T_4 (T_3 ((-1+T_1) T_2 (-1+\alpha_{11}) + (-1+T_2) \alpha_{12}) + (-1+T_3) \alpha_{13}) + (-1+T_4) \alpha_{14}}{T_1 T_2 T_3 T_4} == 0 \ \&\& \ \frac{T_4 (-\alpha_{23} + T_3 (1 - \alpha_{22} + T_2 (-1 + (-1+T_1) \alpha_{21} + \alpha_{22}) + \alpha_{23})) + (-1+T_4)}{T_1 T_2 T_3 T_4}$$

```

t3 = Collect[T1 T2 T3 T4 ucond[[#, 1]], α[_,_], Factor] & /@ {1, 2, 3, 4}
{- (-1 + T1) T2 T3 T4 + (-1 + T1) T2 T3 T4 α11 + (-1 + T2) T3 T4 α12 + (-1 + T3) T4 α13 + (-1 + T4) α14,
- (-1 + T2) T3 T4 + (-1 + T1) T2 T3 T4 α21 + (-1 + T2) T3 T4 α22 + (-1 + T3) T4 α23 + (-1 + T4) α24,
- (-1 + T3) T4 + (-1 + T1) T2 T3 T4 α31 + (-1 + T2) T3 T4 α32 + (-1 + T3) T4 α33 + (-1 + T4) α34,
1 - T4 + (-1 + T1) T2 T3 T4 α41 + (-1 + T2) T3 T4 α42 + (-1 + T3) T4 α43 + (-1 + T4) α44}

{ColumnForm[v = {(-1 + T1) T2 T3 T4, (-1 + T2) T3 T4, (-1 + T3) T4, (-1 + T4)}],
ColumnForm[γ0[A].v], t3 == γ0[A].v - v}

{
(-1 + T1) T2 T3 T4 ,
(-1 + T2) T3 T4
(-1 + T3) T4
-1 + T4

(-1 + T1) T2 T3 T4 α11 + (-1 + T2) T3 T4 α12 + (-1 + T3) T4 α13 + (-1 + T4) α14 , True}
(-1 + T1) T2 T3 T4 α21 + (-1 + T2) T3 T4 α22 + (-1 + T3) T4 α23 + (-1 + T4) α24
(-1 + T1) T2 T3 T4 α31 + (-1 + T2) T3 T4 α32 + (-1 + T3) T4 α33 + (-1 + T4) α34
(-1 + T1) T2 T3 T4 α41 + (-1 + T2) T3 T4 α42 + (-1 + T3) T4 α43 + (-1 + T4) α44

```