Pensieve header: The divisibility condition for $q\Delta$ in Gassner calculus.

Using $\left(\mathbf{T_x\,T_y-1}\right)|(\alpha\text{-}\sigma\text{a})$ get σa=$\boldsymbol{\alpha}$ **– q** $\left(\mathbf{T_x\,T_y-1}\right)$ and then:

$$\texttt{Simplify}\left[\frac{-\sigma\mathbf{a}+\alpha\,\mathbf{Ta}+(-\alpha+\sigma\mathbf{a})\,\mathbf{T_y}}{-1+\mathbf{Ta}}-\sigma\mathbf{a}\ \texttt{/.}\ \left\{\mathbf{Ta}\to\mathbf{T_x\,T_y},\ \sigma\mathbf{a}\to\alpha-\mathbf{q}\left(\mathbf{T_x\,T_y}-1\right)\right\}\right]$$

$$q\,(-1+T_x)\,T_y$$

$$\texttt{Simplify}\left[\frac{-\alpha+\sigma\mathbf{a}\,\mathbf{Ta}+(\alpha-\sigma\mathbf{a})\,\mathbf{T_y}}{-1+\mathbf{Ta}}-\sigma\mathbf{a}\ \texttt{/.}\ \left\{\mathbf{Ta}\to\mathbf{T_x\,T_y},\ \sigma\mathbf{a}\to\alpha-\mathbf{q}\left(\mathbf{T_x\,T_y}-1\right)\right\}\right]$$

$$\mathbf{q}\left(-1+\mathbf{T_y}\right)$$

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dir = SetDirectory["C:/drorbn/AcademicPensieve/2014-06/"];
<< MetaCalculi/MetaCalculi-Program.m
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$$\texttt{Clear}[\alpha,\beta,\gamma,\delta,\theta,\epsilon,\phi,\psi,\Xi,\mu,\omega];$$

$$\gamma0=\Gamma\left[\omega,\ h_a\,\sigma_a+h_b\,\sigma_b+h_S\,\sigma_S,\ \{t_a,t_b,t_S\}.\begin{pmatrix}\sigma_a+\alpha\,(T_a-1) & \beta\,(T_a-1) & \theta\,(T_a-1)\\ \gamma\,(T_b-1) & \sigma_b+\delta\,(T_b-1) & \epsilon\,(T_b-1)\\ \phi\,(T_S-1) & \psi\,(T_S-1) & \sigma_S+\Xi\,(T_S-1)\end{pmatrix}.\{h_a,h_b,h_S\}\right];$$

$$\gamma1=\gamma0\ \texttt{//}\ \texttt{dm}[a,b,c]$$

$$\begin{pmatrix}-\omega\,(-1-\beta+\beta\,T_c) & s_c & \\ s_c & \frac{\gamma+\beta\,\gamma-\alpha\,\delta-\gamma\,T_c-2\,\beta\,\gamma\,T_c+2\,\alpha\,\delta\,T_c+\beta\,\gamma\,T_c^2-\alpha\,\delta\,T_c^2+\delta\,\sigma_a-\delta\,T_c\,\sigma_a+\alpha\,\sigma_b-\alpha\,T_c\,\sigma_b-\sigma_a\,\sigma_b}{-1-\beta+\beta\,T_c} & \frac{(-1+T_c)\ (-\epsilon-\beta}{} \\ s_S & \frac{(-1+T_S)\ (-\phi-\beta\,\phi+\alpha\,\psi+\beta\,\phi\,T_c-\alpha\,\psi\,T_c-\psi\,\sigma_a)}{-1-\beta+\beta\,T_c} & \frac{\Xi+\beta\,\Xi-\theta\,\psi-\beta\,\Xi\,T_c+\theta\,\psi\,T_c-\Xi\,T_S-\beta\,\Xi}{} \\ \Sigma & \sigma_a\,\sigma_b & \end{pmatrix}$$

$$\gamma1[A]-\texttt{DiagonalMatrix}[\{\sigma_a\,\sigma_b,\ \sigma_S\}]\ \texttt{//}\ \texttt{Simplify}\ \texttt{//}\ \texttt{MatrixForm}$$

$$\begin{pmatrix}\frac{(-1+T_c)\ (-\gamma-\beta\,\gamma+\alpha\,\delta+(\beta\,\gamma-\alpha\,\delta)\,T_c-\alpha\,\sigma_b-\sigma_a\,(\delta+\beta\,\sigma_b))}{-1-\beta+\beta\,T_c} & \frac{(-1+T_c)\ (-\epsilon-\beta\,\epsilon+\delta\,\theta+(\beta\,\epsilon-\delta\,\theta)\,T_c-\theta\,\sigma_b)}{-1-\beta+\beta\,T_c}\\ \frac{(-1+T_S)\ (-\phi-\beta\,\phi+\alpha\,\psi+(\beta\,\phi-\alpha\,\psi)\,T_c-\psi\,\sigma_a)}{-1-\beta+\beta\,T_c} & \frac{(-(1+\beta)\,\Xi+\theta\,\psi+(\beta\,\Xi-\theta\,\psi)\,T_c)\ (-1+T_S)}{-1-\beta+\beta\,T_c}\end{pmatrix}$$

$$\texttt{Clear}[\alpha,\theta,\phi,\Xi,\omega];$$

$$\gamma0=\Gamma\left[\omega,\ h_a\,\sigma_a+h_S\,\sigma_S,\ \{t_a,t_S\}.\begin{pmatrix}\sigma_a+\alpha\,(T_a-1) & \theta\,(T_a-1)\\ \phi\,(T_S-1) & \sigma_S+\Xi\,(T_S-1)\end{pmatrix}.\{h_a,h_S\}\right];$$

$$\gamma1=\gamma0\ \texttt{//}\ \texttt{dS}[a]$$

$$\begin{pmatrix}\frac{\omega\,(\alpha-\alpha\,T_a+T_a\,\sigma_a)}{T_a\,\sigma_a} & s_a & s_S\\ s_a & -\frac{T_a}{-\alpha+\alpha\,T_a-T_a\,\sigma_a} & \frac{\theta\,(-1+T_a)}{-\alpha+\alpha\,T_a-T_a\,\sigma_a}\\ s_S & \frac{\phi\,T_a\,(-1+T_S)}{-\alpha+\alpha\,T_a-T_a\,\sigma_a} & \frac{\alpha\,\Xi-\theta\,\phi-\alpha\,\Xi\,T_a+\theta\,\phi\,T_a-\alpha\,\Xi\,T_S+\theta\,\phi\,T_S+\alpha\,\Xi\,T_a\,T_S-\theta\,\phi\,T_a\,T_S+\Xi\,T_a\,\sigma_a-\Xi\,T_a\,T_S\,\sigma_a-\alpha\,\sigma_S+\alpha\,T_a\,\sigma_S-T_a\,\sigma_a\,\sigma_S}{-\alpha+\alpha\,T_a-T_a\,\sigma_a}\\ \Sigma & \frac{1}{\sigma_a} & \sigma_S\end{pmatrix}$$

$$\gamma1[A]-\texttt{DiagonalMatrix}\left[\left\{\frac{1}{\sigma_a},\ \sigma_S\right\}\right]\ \texttt{//}\ \texttt{Simplify}\ \texttt{//}\ \texttt{MatrixForm}$$

$$\begin{pmatrix}\frac{\alpha\,(-1+T_a)}{\sigma_a\,(\alpha+T_a\,(-\alpha+\sigma_a))} & \frac{\theta\,(-1+T_a)}{-\alpha+T_a\,(\alpha-\sigma_a)}\\ \frac{\phi\,T_a\,(-1+T_S)}{-\alpha+T_a\,(\alpha-\sigma_a)} & \frac{(-1+T_S)\ (-\alpha\,\Xi+\theta\,\phi+T_a\,(\alpha\,\Xi-\theta\,\phi-\Xi\,\sigma_a))}{-\alpha+T_a\,(\alpha-\sigma_a)}\end{pmatrix}$$

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Clear[α, θ, ϕ, Ξ, ω];
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$$\gamma 0 = \Gamma\left[\omega,\ h_a\,\sigma_a + h_S\,\sigma_S,\ \{t_a, t_S\}.\begin{pmatrix} \sigma_a + \alpha\,(T_a - 1) & \theta\,(T_a - 1) \\ \phi\,(T_S - 1) & \sigma_S + \Xi\,(T_S - 1) \end{pmatrix}.\{h_a, h_S\}\right];$$

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γ1 = γ0 // qΔ[a, b, c]
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$$\begin{pmatrix} \omega & s_b & s_c & s_S \\ s_b & -\alpha\,T_c + \alpha\,T_b\,T_c + \sigma_a & \alpha\,(-1 + T_b)\,T_c & \theta\,(-1 + T_b)\,T_c \\ s_c & \alpha\,(-1 + T_c) & -\alpha + \alpha\,T_c + \sigma_a & \theta\,(-1 + T_c) \\ s_S & \phi\,(-1 + T_S) & \phi\,(-1 + T_S) & -\Xi + \Xi\,T_S + \sigma_S \\ \Sigma & \sigma_a & \sigma_a & \sigma_S \end{pmatrix}$$

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γ1[A] - DiagonalMatrix[{σa , σa, σS}] // Simplify // MatrixForm
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$$\begin{pmatrix} \alpha\,(-1 + T_b)\,T_c & \alpha\,(-1 + T_b)\,T_c & \theta\,(-1 + T_b)\,T_c \\ \alpha\,(-1 + T_c) & \alpha\,(-1 + T_c) & \theta\,(-1 + T_c) \\ \phi\,(-1 + T_S) & \phi\,(-1 + T_S) & \Xi\,(-1 + T_S) \end{pmatrix}$$