Pensieve Header: Cheat sheat β verification program, continues pensieve://2014-05/.

Program

```
<< KnotTheory`
```

```
Loading KnotTheory` version of April 3, 2014, 16:23:56.0784.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
βSimplify = Simplify;
SetAttributes[βCollect, Listable];
βCollect[B[ω_, σ_, μ_]] := B[
   βSimplify[ω], σ,
   Collect[μ, _h, Collect[#, _t, βSimplify] &]
  ];
hL[b_] := Union[Cases[b, h[s_] ⧴ s, Infinity]];
tL[b_] := Union[Cases[b, t[s_] | T_s_ ⧴ s, Infinity]];
dL[b_] := Union[hL[b], tL[b]];
σ_ ⊢ h_ := (∂_h σ /. 0 → 1);
B[ω_, σ_, μ_]@A := Module[
   {tails, heads},
   tails = tL[B[ω, σ, μ]]; heads = hL[B[ω, σ, μ]];
   Outer[βSimplify[∂_{t[#1],h[#2]} μ] &, tails, heads]
  ];
βForm[B[ω_, σ_, μ_]] := Module[
   {tails, heads, mat},
   tails = tL[B[ω, σ, μ]]; heads = hL[B[ω, σ, μ]];
   mat = Outer[βSimplify[∂_{h[#1],t[#2]} μ] &, heads, tails];
   PrependTo[mat, t /@ tails];
   mat = Join[
     {Prepend[h /@ heads, ω]},
     Transpose[mat],
     {Prepend[(σ ⊢ h[#]) & /@ heads, "1+Σ/ω"]}
    ];
   MatrixForm[mat]
  ];
βForm[else_] := else /. b_B ⧴ βForm[b];
Format[b_B, StandardForm] := βForm[b];
B /: B[ω1_, σ1_, μ1_] == B[ω2_, σ2_, μ2_] := Module[
   {heads, tails},
   tails = tL[{B[ω1, σ1, μ1], B[ω2, σ2, μ2]}];
   heads = hL[{B[ω1, σ1, μ1], B[ω2, σ2, μ2]}];
   (ω1 == ω2) && (σ1 == σ2) && (
     And @@ Flatten[Outer[
        (Coefficient[μ1, t[#1] h[#2]] == Coefficient[μ2, t[#1] h[#2]]) &,
        tails, heads
       ]]
    )
```

```
  ];

B /: B[ω1_, σ1_, μ1_] B[ω2_, σ2_, μ2_] := B[ω1 * ω2, σ1 + σ2, ω2 μ1 + ω1 μ2];
tm[x_, y_, z_][b_] := b /. {t[x] → t[z], t[y] → t[z], T_x → T_z, T_y → T_z};
hm[x_, y_, z_][B[ω_, σ_, μ_]] := B[ω,
    h[z] (σ ⊢ h[x]) (σ ⊢ h[y]) + (σ /. h[x] | h[y] → 0),
    h[z] (D[μ, h[x]] + (σ ⊢ h[x]) ∂_h[y] μ) + (μ /. h[x] | h[y] → 0)
   ] // βCollect;
swapth[y_, x_][B[ω_, σ_, μ_]] := Module[
   {α, β, γ, δ},
   ( α  β )   ( Coefficient[μ, t[y] h[x]]   D[μ, t[y]] /. h[x] → 0 )
   ( γ  δ ) = (   D[μ, h[x]] /. t[y] → 0     μ /. h[x] | t[y] → 0   );
   B[ω + α, σ, {(σ ⊢ h[x]) t[y], 1}.( α   β                     ).{h[x], 1}] // βCollect
                                    ( γ   ((ω + α) δ - γ * β) / ω )
  ];
dm0[x_, y_, z_][b_] := b // swapth[x, y] // hm[x, y, z] // tm[x, y, z];
dm[a_, b_, c_][B[ω0_, σ_, μ_]] := Module[
  {ω, α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ, σa, σb},
  ω = ω0 /. {T_a → T_c, T_b → T_c};
  {σa, σb} = {σ ⊢ h[a], σ ⊢ h[b]} /. {T_a → T_c, T_b → T_c};
  ( α  β  θ )
  ( γ  δ  ϵ ) =
  ( ϕ  ψ  Ξ )
  (  ∂_t[a],h[a] μ                     ∂_t[a],h[b] μ                     ∂_t[a] μ /. h[a] | h[b] → 0
     ∂_t[b],h[a] μ                     ∂_t[b],h[b] μ                     ∂_t[b] μ /. h[a] | h[b] → 0
     ∂_h[a] μ /. t[a] | t[b] → 0       ∂_h[b] μ /. t[a] | t[b] → 0       μ /. t[a] | t[b] | h[a] | h[b] →
  
   ) /. {T_a → T_c, T_b → T_c};
  B[ω + β,
    h[c] σa σb + (σ /. h[a] | h[b] → 0 /. {T_a → T_c, T_b → T_c}),
    {t[c], 1}.( γ + σa δ + σb (α + σa β) + (β γ - α δ)/ω    ϵ + σb θ + (β ϵ - δ θ)/ω ).{h[c], 1}
              ( ϕ + σa ψ + (β ϕ - α ψ)/ω                    Ξ + (β Ξ - ψ θ)/ω        )
   ] // βCollect
 ]

Unprotect[NonCommutativeMultiply];
b1_B ** b2_B := Module[
   {ρ, σ, labels},
   ρ = b1 * (b2 /. {h[s_] ⧴ h[σ[s]], t[s_] ⧴ t[σ[s]], T_s_ ⧴ T_σ[s]});
   labels = dL[{b1, b2}];
   Do[ρ = ρ // dm[s, σ[s], s], {s, labels}];
   ρ
  ];
```

```
βbRp[x_, y_] := B[1, T_x h[y], (T_x - 1) * t[x] h[y]];
βbRm[x_, y_] := B[1, h[y] / T_x, (1 / T_x - 1) * t[x] h[y]];

βbZ[L_] := Module[{s, Z, c, k},
  s = Skeleton[L];
  Z = Times @@ PD[L] /.
    X[i_, j_, k_, l_] :> If[PositiveQ[X[i, j, k, l]], βbRp[l, i], βbRm[j, i]];
  Do[Z = Z // dm[s⟦c, 1⟧, s⟦c, k⟧, s⟦c, 1⟧], {c, Length[s]},
   {k, 2, Length[s⟦c⟧]}];
  Z]
```

R3 for β-better

```
{βbRp[1, 2], βbRm[1, 2]}
```

$$\left\{\begin{pmatrix} 1 & h[2] \\ t[1] & -1+T_1 \\ 1+\Sigma/\omega & T_1 \end{pmatrix}, \begin{pmatrix} 1 & h[2] \\ t[1] & -1+\frac{1}{T_1} \\ 1+\Sigma/\omega & \frac{1}{T_1} \end{pmatrix}\right\}$$

```
βbRp[1, 2] ** βbRp[1, 3]
```

$$\begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & 0 & -1+T_1 & -1+T_1 \\ 1+\Sigma/\omega & 1 & T_1 & T_1 \end{pmatrix}$$

```
{βbRp[1, 2] ** βbRp[1, 3] ** βbRp[2, 3], βbRp[2, 3] ** βbRp[1, 3] ** βbRp[1, 2]}
```

$$\left\{\begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & 0 & -1+T_1 & -1+T_1 \\ t[2] & 0 & 0 & T_1(-1+T_2) \\ 1+\Sigma/\omega & 1 & T_1 & T_1 T_2 \end{pmatrix}, \begin{pmatrix} 1 & h[1] & h[2] & h[3] \\ t[1] & 0 & -1+T_1 & -1+T_1 \\ t[2] & 0 & 0 & T_1(-1+T_2) \\ 1+\Sigma/\omega & 1 & T_1 & T_1 T_2 \end{pmatrix}\right\}$$

dm for β-better

```
{B0 =
  B[ω, {σa, σb, σ}.{h@a, h@b, h@S}, {t@a, t@b, t@S}.(α β θ; γ δ ϵ; ϕ ψ Ξ).{h@a, h@b, h@S}],
 B0 // dm[a, b, c]}
```

$$\left\{\begin{pmatrix} \omega & h[a] & h[b] & h[S] \\ t[a] & \alpha & \beta & \theta \\ t[b] & \gamma & \delta & \epsilon \\ t[S] & \phi & \psi & \Xi \\ 1+\Sigma/\omega & \sigma_a & \sigma_b & \sigma \end{pmatrix}, \begin{pmatrix} \beta+\omega & h[c] & h[S] \\ t[c] & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ t[S] & \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \\ 1+\Sigma/\omega & \sigma_a\sigma_b & \sigma \end{pmatrix}\right\}$$

```
(B0 // swapth[a, b] // hm[a, b, c] // tm[a, b, c]) == (B0 // dm[a, b, c]) // Simplify
```

True

$$\begin{pmatrix} \beta+\omega & h[c] & h[S] \\ t[c] & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ t[S] & \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \\ \text{"}1+\Sigma/\omega\text{"} & \sigma_a\sigma_b & \sigma \end{pmatrix} \text{ // FullSimplify // MatrixForm}$$

$$\begin{pmatrix} \beta+\omega & h[c] & h[S] \\ t[c] & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{-\delta\theta+\epsilon(\beta+\omega)+\theta\omega\sigma_b}{\omega} \\ t[S] & \frac{-\alpha\psi+\phi(\beta+\omega)+\psi\omega\sigma_a}{\omega} & \Xi+\frac{\beta\Xi-\theta\psi}{\omega} \\ 1+\Sigma/\omega & \sigma_a\sigma_b & \sigma \end{pmatrix}$$

Back to β (and β-Burau)

$$B0 = B\left[\omega,\ \{\sigma_a, \sigma_b, \sigma\}.\{h@a, h@b, h@S\},\ \{t@a, t@b, t@S\}.\begin{pmatrix} \alpha & \beta & \theta \\ \gamma & \delta & \epsilon \\ \phi & \psi & \Xi \end{pmatrix}.\{h@a, h@b, h@S\}\right]$$

$$\begin{pmatrix} \omega & h[a] & h[b] & h[S] \\ t[a] & \alpha & \beta & \theta \\ t[b] & \gamma & \delta & \epsilon \\ t[S] & \phi & \psi & \Xi \\ 1+\Sigma/\omega & \sigma_a & \sigma_b & \sigma \end{pmatrix}$$

```
hm[a, b, c][B0]@A // MatrixForm
```

$$\begin{pmatrix} \alpha+\beta\sigma_a & \theta \\ \gamma+\delta\sigma_a & \epsilon \\ \phi+\psi\sigma_a & \Xi \end{pmatrix}$$

hm and swapth

$$\Big(\omega^{-1}\,hm[a, b, c][B0]@A\ /.\ \text{Thread}[\{\alpha, \beta, \gamma, \delta, \theta, \epsilon, \phi, \psi, \Xi\} \to \omega\{\alpha, \beta, \gamma, \delta, \theta, \epsilon, \phi, \psi, \Xi\}]\Big) \text{ // FullSimplify // MatrixForm}$$

$$\begin{pmatrix} \alpha+\beta\sigma_a & \theta \\ \gamma+\delta\sigma_a & \epsilon \\ \phi+\psi\sigma_a & \Xi \end{pmatrix}$$

```
((ω + α)^-1 swapth[a, a][B0]@A /.
    Thread[{α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ} → ω {α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ}]) //
  FullSimplify // MatrixForm
```

$$\begin{pmatrix} \frac{\alpha\sigma_a}{1+\alpha} & \frac{\beta\sigma_a}{1+\alpha} & \frac{\theta\sigma_a}{1+\alpha} \\ \frac{\gamma}{1+\alpha} & -\frac{\beta\gamma}{1+\alpha}+\delta & \epsilon-\frac{\gamma\theta}{1+\alpha} \\ \frac{\phi}{1+\alpha} & -\frac{\beta\phi}{1+\alpha}+\psi & \Xi-\frac{\theta\phi}{1+\alpha} \end{pmatrix}$$

```
(1 + α) ((ω + α)^-1 swapth[a, a][B0]@A /. Thread[{α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ} →
      ω {α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ}]) // FullSimplify // MatrixForm
```

$$\begin{pmatrix} \alpha\sigma_a & \beta\sigma_a & \theta\sigma_a \\ \gamma & -\beta\gamma+\delta+\alpha\delta & \epsilon+\alpha\epsilon-\gamma\theta \\ \phi & -\beta\phi+\psi+\alpha\psi & \Xi+\alpha\Xi-\theta\phi \end{pmatrix}$$

dm

```
((ω + β)^-1 ( γ + (β γ-α δ)/ω + δ σa + (α + β σa) σb    (β ϵ-δ θ+ϵ ω+θ ω σb)/ω
              (β ϕ-α ψ+ϕ ω+ψ ω σa)/ω                   (β Ξ-θ ψ+Ξ ω)/ω        ) /.
    Thread[{α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ} → ω {α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ}]) //
  FullSimplify // MatrixForm
```

$$\begin{pmatrix} \frac{\gamma+\beta\gamma-\alpha\delta+\alpha\sigma_b+\sigma_a(\delta+\beta\sigma_b)}{1+\beta} & \frac{\epsilon+\beta\epsilon-\delta\theta+\theta\sigma_b}{1+\beta} \\ \frac{\phi+\beta\phi-\alpha\psi+\psi\sigma_a}{1+\beta} & \Xi-\frac{\theta\psi}{1+\beta} \end{pmatrix}$$

```
( (γ+β γ-α δ+α σb+σa (δ+β σb))/(1+β)   (ϵ+β ϵ-δ θ+θ σb)/(1+β)
  (ϕ+β ϕ-α ψ+ψ σa)/(1+β)               Ξ - (θ ψ)/(1+β)        ) - ( γ ϵ
                                                                    ϕ Ξ ) // FullSimplify // MatrixForm
```

$$\begin{pmatrix} \frac{\alpha(-\delta+\sigma_b)+\sigma_a(\delta+\beta\sigma_b)}{1+\beta} & \frac{\theta(-\delta+\sigma_b)}{1+\beta} \\ \frac{\psi(-\alpha+\sigma_a)}{1+\beta} & -\frac{\theta\psi}{1+\beta} \end{pmatrix}$$

```
Plus[( (γ+β γ-α δ+α σb+σa (δ+β σb))/(1+β)   (ϵ+β ϵ-δ θ+θ σb)/(1+β)
       (ϕ+β ϕ-α ψ+ψ σa)/(1+β)               Ξ - (θ ψ)/(1+β)        ) /. {α → α + σa, δ → δ + σb, Ξ → Ξ + σ},
    ( -σa σb   0
       0      -σ )] // FullSimplify // MatrixForm
```

$$\begin{pmatrix} \gamma-\frac{\alpha\delta}{1+\beta} & \epsilon-\frac{\delta\theta}{1+\beta} \\ \phi-\frac{\alpha\psi}{1+\beta} & \Xi-\frac{\theta\psi}{1+\beta} \end{pmatrix}$$

$$(1+\beta)\begin{pmatrix}\gamma-\frac{\alpha\,\delta}{1+\beta} & \epsilon-\frac{\delta\,\theta}{1+\beta}\\ \phi-\frac{\alpha\,\psi}{1+\beta} & \Xi-\frac{\theta\,\psi}{1+\beta}\end{pmatrix}\ \text{// FullSimplify // MatrixForm}$$

$$\begin{pmatrix}\gamma+\beta\,\gamma-\alpha\,\delta & \epsilon+\beta\,\epsilon-\delta\,\theta\\ \phi+\beta\,\phi-\alpha\,\psi & \Xi+\beta\,\Xi-\theta\,\psi\end{pmatrix}$$

$$\text{Plus}\left[\begin{pmatrix}\frac{\gamma+\beta\,\gamma-\alpha\,\delta+\alpha\,\sigma_b+\sigma_a\,(\delta+\beta\,\sigma_b)}{1+\beta} & \frac{\epsilon+\beta\,\epsilon-\delta\,\theta+\theta\,\sigma_b}{1+\beta}\\ \frac{\phi+\beta\,\phi-\alpha\,\psi+\psi\,\sigma_a}{1+\beta} & \Xi-\frac{\theta\,\psi}{1+\beta}\end{pmatrix}\ /.\ \{\alpha\to\alpha+\sigma_a-1,\ \delta\to\delta+\sigma_b-1,\ \Xi\to\Xi+\sigma-1\},\right.$$
$$\left.\begin{pmatrix}1-\sigma_a\,\sigma_b & 0\\ 0 & 1-\sigma\end{pmatrix}\right]\ \text{// FullSimplify // MatrixForm}$$

$$\begin{pmatrix}\frac{\alpha+\beta+\gamma+\beta\,\gamma+\delta-\alpha\,\delta}{1+\beta} & \epsilon+\frac{\theta-\delta\,\theta}{1+\beta}\\ \phi+\frac{\psi-\alpha\,\psi}{1+\beta} & \Xi-\frac{\theta\,\psi}{1+\beta}\end{pmatrix}$$

$$\text{Plus}\left[\begin{pmatrix}\frac{\gamma+\beta\,\gamma-\alpha\,\delta+\alpha\,\sigma_b+\sigma_a\,(\delta+\beta\,\sigma_b)}{1+\beta} & \frac{\epsilon+\beta\,\epsilon-\delta\,\theta+\theta\,\sigma_b}{1+\beta}\\ \frac{\phi+\beta\,\phi-\alpha\,\psi+\psi\,\sigma_a}{1+\beta} & \Xi-\frac{\theta\,\psi}{1+\beta}\end{pmatrix}\ /.\ \{\alpha\to\alpha-1,\ \delta\to\delta-1,\ \Xi\to\Xi-1\},\ \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right]\ \text{//}$$
$$\text{FullSimplify // MatrixForm}$$

$$\begin{pmatrix}\frac{\alpha+\beta+\gamma+\beta\,\gamma+\delta-\alpha\,\delta+(-1+\alpha)\,\sigma_b+\sigma_a\,(-1+\delta+\beta\,\sigma_b)}{1+\beta} & \frac{\epsilon+\beta\,\epsilon+\theta-\delta\,\theta+\theta\,\sigma_b}{1+\beta}\\ \frac{\phi+\beta\,\phi+\psi-\alpha\,\psi+\psi\,\sigma_a}{1+\beta} & \Xi-\frac{\theta\,\psi}{1+\beta}\end{pmatrix}$$

Alexander with β-better

```
{Knot[8, 17] // βbZ, Alexander[Knot[8, 17]][T1] // βSimplify}
```

KnotTheory::loading : Loading precomputed data in PD4Knots`.

$$\left\{\begin{pmatrix}-8-\frac{1}{T_1^2}+\frac{4}{T_1}+11\,T_1-8\,T_1^2+4\,T_1^3-T_1^4 & h[1]\\ t[1] & 0\\ 1+\Sigma/\omega & 1\end{pmatrix},\ 11-\frac{1}{T_1^3}+\frac{4}{T_1^2}-\frac{8}{T_1}-8\,T_1+4\,T_1^2-T_1^3\right\}$$

The MVA with β-better

```
βbMVA[L_Link] := Module[{Hs, ω, σ, μ, A, M},
  {ω, σ, μ} = List @@ βbZ[L];
  Hs = Rest[h /@ (First /@ Skeleton[L])];
  A = Outer[Coefficient[μ, #1 * #2] &, Hs, Hs /. h[a_] :> t[a]];
  M = A - ω DiagonalMatrix[((σ ⊢ #) - 1) & /@ Hs];
  Factor[ω^(2-Length@Skeleton@L) Det[M] / (1 - T_(Skeleton[L][[1,1]]))]
]
```

```
Link["L6a4"] // βbZ
```

KnotTheory::loading : Loading precomputed data in PD4Links`.

$$\left(\begin{array}{ccc} \frac{(T_1(-1+T_5)(-1+T_9)-T_5(-1+T_9)+T_9)((-1+T_5)(-1+T_9)+T_1(-1+T_5+T_9))}{T_1 T_5 T_9} & h[1] & \\ t[1] & \frac{(-1+T_1)(-1+T_5)(T_1(-1+T_5)+T_5(-1+T_9))(-1+T_9)}{T_1 T_5 T_9} & -(\\ t[5] & -\frac{(1+T_1(-1+T_5))(-1+T_5)(-1+T_9)}{T_5 T_9} & (-1+T_1) \\ t[9] & -\frac{(-1+T_5)(1+T_1(-2+T_9)-T_9)(-1+T_9)}{T_1 T_9} & \\ 1+\Sigma/\omega & 1 & \end{array}\right.$$

```
βbMVA[Link["L6a4"]]
```

$$-\frac{(-1+T_1)(-1+T_5)(-1+T_9)}{T_1 T_5}$$

```
Factor[1/βbMVA[#] (MultivariableAlexander[#][T] /. T[i_] :> T_(Skeleton[#][[i,1]]))] & /@
 AllLinks[{2, 8}]
```

KnotTheory::loading : Loading precomputed data in MultivariableAlexander4Links`.

$$\left\{T_1^2 T_3,\ T_1^{3/2} T_5^{3/2},\ \sqrt{T_1}\ T_5^{3/2},\ T_1^{3/2}\sqrt{T_5},\ T_1^2 T_7^2,\ T_1^2 T_7^2,\ -\frac{\sqrt{T_1}\sqrt{T_5}}{\sqrt{T_9}},\ -T_1^{3/2} T_5^{3/2} T_9^{3/2},\right.$$
$$-\frac{\sqrt{T_1}\sqrt{T_5}}{T_9^{3/2}},\ \sqrt{T_1}\sqrt{T_5},\ T_1^{3/2} T_5^{7/2},\ \frac{\sqrt{T_1}}{T_5^{3/2}},\ \frac{\sqrt{T_1}}{T_5^{3/2}},\ T_1 T_7^2,\ \frac{1}{T_7},\ -\frac{T_1^{3/2}\sqrt{T_5}}{\sqrt{T_9}},\ T_1^{3/2} T_5^{7/2},$$
$$\sqrt{T_1}\ T_5^{5/2},\ \sqrt{T_1}\ T_5^{3/2},\ \frac{\sqrt{T_1}}{\sqrt{T_5}},\ T_1^{3/2} T_5^{3/2},\ \sqrt{T_1}\ T_5^{3/2},\ \frac{T_1^{3/2}}{\sqrt{T_5}},\ \frac{T_1^{3/2}}{\sqrt{T_5}},\ T_1^{3/2} T_5^{7/2},\ \frac{T_1}{T_7},\ T_1 T_7,$$
$$T_1^2 T_7^3,\ T_1^2 T_7^3,\ T_1^{5/2} T_9^{5/2},\ T_1^{5/2} T_9^{5/2},\ T_1^{5/2} T_9^{5/2},\ -T_1^{3/2} T_5^{3/2}\sqrt{T_9},\ -\sqrt{T_1},\ -T_1^{3/2} T_5^2 T_{11}^2,$$
$$-\frac{\sqrt{T_1}}{T_{11}^2},\ -\frac{\sqrt{T_1}\ T_5}{T_{11}},\ -\frac{T_1^{3/2}\sqrt{T_5}}{T_{13}^{3/2}},\ T_1^{3/2} T_5^{3/2} T_9^{3/2} T_{13}^{3/2},\ T_1^{3/2} T_5^{3/2},\ \sqrt{T_1}\sqrt{T_5},\ -T_1^{3/2} T_5^2 T_{11}^2,$$
$$\left.-\sqrt{T_1}\ T_5^2 T_{11},\ -\sqrt{T_1}\ T_5^{3/2} T_{11}^{3/2},\ -T_1^{3/2} T_5^{5/2}\sqrt{T_{13}},\ \frac{\sqrt{T_1}\sqrt{T_5}}{\sqrt{T_9}\sqrt{T_{13}}},\ \sqrt{T_1}\sqrt{T_5}\sqrt{T_9}\sqrt{T_{13}}\right\}$$

Burau Calculus

```
Plus[
```
$$\left(\begin{array}{ccc} \beta+\omega & h[c] & h[S] \\ t[c] & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ t[S] & \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \\ \text{"}1+\Sigma/\omega\text{"} & \sigma_a\sigma_b & \sigma \end{array}\right) /.$$
$$\{\alpha\to\alpha+\omega\sigma_a,\ \delta\to\delta+\omega\sigma_b,\ \Xi\to\Xi+\omega\sigma\},$$
$$-\left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & \omega\sigma_a\sigma_b & 0 \\ 0 & 0 & \omega\sigma \\ 0 & 0 & 0 \end{array}\right)$$
```
] // FullSimplify // MatrixForm
```

$$\left(\begin{array}{ccc} \beta+\omega & h[c] & h[S] \\ t[c] & \frac{-\alpha\delta+\gamma(\beta+\omega)+\beta\omega\sigma_a\sigma_b}{\omega} & \epsilon+\frac{\beta\epsilon-\delta\theta}{\omega} \\ t[S] & \phi+\frac{\beta\phi-\alpha\psi}{\omega} & \Xi+\beta\sigma+\frac{\beta\Xi-\theta\psi}{\omega} \\ 1+\Sigma/\omega & \sigma_a\sigma_b & \sigma \end{array}\right)$$

```
Plus[
```
$$\left(\begin{array}{ccc} \beta+\omega & h[c] & h[S] \\ t[c] & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\sigma_a+(\alpha+\beta\sigma_a)\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ t[S] & \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \\ \text{"}1+\Sigma/\omega\text{"} & \sigma_a\sigma_b & \sigma \end{array}\right) /.$$
$$\{\alpha\to\alpha+\omega(\sigma_a-1),\ \delta\to\delta+\omega(\sigma_b-1),\ \Xi\to\Xi+\omega(\sigma-1)\},$$
$$-\left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & \omega(\sigma_a\sigma_b-1) & 0 \\ 0 & 0 & \omega(\sigma-1) \\ 0 & 0 & 0 \end{array}\right)$$
```
] // FullSimplify // MatrixForm
```

$$\left(\begin{array}{ccc} \beta+\omega & h[c] & h[S] \\ t[c] & \alpha+\gamma+\delta+\frac{\beta\gamma-\alpha\delta}{\omega}+\beta\sigma_a\sigma_b & \epsilon+\theta+\frac{\beta\epsilon-\delta\theta}{\omega} \\ t[S] & \phi+\psi+\frac{\beta\phi-\alpha\psi}{\omega} & \Xi+\beta(-1+\sigma)+\frac{\beta\Xi-\theta\psi}{\omega} \\ 1+\Sigma/\omega & \sigma_a\sigma_b & \sigma \end{array}\right)$$

Plus[

$$\begin{pmatrix} \beta+\omega & \mathbf{h[c]} & \mathbf{h[S]} \\ \mathbf{t[c]} & \gamma+\frac{\beta\gamma-\alpha\delta}{\omega}+\delta\,\sigma_a+(\alpha+\beta\,\sigma_a)\,\sigma_b & \frac{\beta\epsilon-\delta\theta+\epsilon\omega+\theta\omega\sigma_b}{\omega} \\ \mathbf{t[S]} & \frac{\beta\phi-\alpha\psi+\phi\omega+\psi\omega\sigma_a}{\omega} & \frac{\beta\Xi-\theta\psi+\Xi\omega}{\omega} \\ \text{"}1+\Sigma/\omega\text{"} & \sigma_a\,\sigma_b & \sigma \end{pmatrix} \;/.$$

$\{\alpha\to\alpha-\omega,\ \delta\to\delta-\omega,\ \Xi\to\Xi-\omega\}$,

$$+\begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \\ 0 & 0 & 0 \end{pmatrix}$$

] // FullSimplify // MatrixForm

$$\begin{pmatrix} \beta+\omega & \mathrm{h[c]} & \mathrm{h[S]} \\ \mathrm{t[c]} & \frac{\beta\gamma-\alpha\delta+(\alpha+\gamma+\delta)\,\omega+(\alpha-\omega)\,\omega\,\sigma_b+\omega\,\sigma_a\,(\delta-\omega+\beta\,\sigma_b)}{\omega} & \epsilon+\theta+\frac{\beta\epsilon-\delta\theta}{\omega}+\theta\,\sigma_b \\ \mathrm{t[S]} & \phi+\psi+\frac{\beta\phi-\alpha\psi}{\omega}+\psi\,\sigma_a & -\beta+\Xi+\frac{\beta\Xi-\theta\psi}{\omega} \\ 1+\Sigma/\omega & \sigma_a\,\sigma_b & \sigma \end{pmatrix}$$

The Dvisibility Condition C_2

```
dir = SetDirectory["C:/drorbn/AcademicPensieve/2014-06/"];
<< MetaCalculi/MetaCalculi-Program.m
```

```
Clear[α, β, γ, δ, θ, ϵ, ϕ, ψ, Ξ, μ, ω];
```

$\gamma 0 = \Gamma\Big[\omega,\ h_a\,\sigma_a+h_b\,\sigma_b+h_S\,\sigma_S,$

$$\{t_a, t_b, t_S\}.\begin{pmatrix} \sigma_a+\alpha\,(T_a-1) & \beta\,(T_a-1) & \theta\,(T_a-1) \\ \gamma\,(T_b-1) & \sigma_b+\delta\,(T_b-1) & \epsilon\,(T_b-1) \\ \phi\,(T_S-1) & \psi\,(T_S-1) & \sigma_S+\Xi\,(T_S-1) \end{pmatrix}.\{h_a, h_b, h_S\}\Big];$$

```
γ1 = γ0 // dm[a, b, c]
```

$$\begin{pmatrix} -\omega\,(-1-\beta+\beta\,T_c) & s_c & \\ s_c & \frac{\gamma+\beta\gamma-\alpha\delta-\gamma T_c-2\beta\gamma T_c+2\alpha\delta T_c+\beta\gamma T_c^2-\alpha\delta T_c^2+\delta\sigma_a-\delta T_c\sigma_a+\alpha\sigma_b-\alpha T_c\sigma_b-\sigma_a\sigma_b}{-1-\beta+\beta T_c} & \frac{(-1+T_c)\,(-\epsilon-\ldots}{\ldots} \\ s_S & \frac{(-1+T_S)\,(-\phi-\beta\phi+\alpha\psi+\beta\phi T_c-\alpha\psi T_c-\psi\sigma_a)}{-1-\beta+\beta T_c} & \frac{\Xi+\beta\Xi-\theta\psi-\beta\Xi T_c+\theta\psi T_c-\Xi T_S-\beta\Xi\ldots}{\ldots} \\ \Sigma & \sigma_a\,\sigma_b & \end{pmatrix}$$

```
γ1[A] - DiagonalMatrix[{σa σb, σS}] // Simplify // MatrixForm
```

$$\begin{pmatrix} \frac{(-1+T_c)\,(-\gamma-\beta\gamma+\alpha\delta+(\beta\gamma-\alpha\delta)\,T_c-\alpha\,\sigma_b-\sigma_a\,(\delta+\beta\,\sigma_b))}{-1-\beta+\beta\,T_c} & \frac{(-1+T_c)\,(-\epsilon-\beta\epsilon+\delta\theta+(\beta\epsilon-\delta\theta)\,T_c-\theta\,\sigma_b)}{-1-\beta+\beta\,T_c} \\ \frac{(-1+T_S)\,(-\phi-\beta\phi+\alpha\psi+(\beta\phi-\alpha\psi)\,T_c-\psi\,\sigma_a)}{-1-\beta+\beta\,T_c} & \frac{(-(1+\beta)\,\Xi+\theta\psi+(\beta\Xi-\theta\psi)\,T_c)\,(-1+T_S)}{-1-\beta+\beta\,T_c} \end{pmatrix}$$

Clear[α, θ, ϕ, Ξ, ω];

$\gamma 0 = \Gamma\left[\omega,\ h_a\,\sigma_a + h_S\,\sigma_S,\ \{t_a, t_S\}.\begin{pmatrix} \sigma_a + \alpha\,(T_a - 1) & \theta\,(T_a - 1) \\ \phi\,(T_S - 1) & \sigma_S + \Xi\,(T_S - 1) \end{pmatrix}.\{h_a, h_S\}\right];$

$\gamma 1 = \gamma 0$ // **dS**[a]

$$\begin{pmatrix} \frac{\omega\,(\alpha-\alpha\,T_a+T_a\,\sigma_a)}{T_a\,\sigma_a} & s_a & s_S \\ s_a & -\frac{T_a}{-\alpha+\alpha\,T_a-T_a\,\sigma_a} & \frac{\theta\,(-1+T_a)}{-\alpha+\alpha\,T_a-T_a\,\sigma_a} \\ s_S & \frac{\phi\,T_a\,(-1+T_S)}{-\alpha+\alpha\,T_a-T_a\,\sigma_a} & \frac{\alpha\,\Xi-\theta\,\phi-\alpha\,\Xi\,T_a+\theta\,\phi\,T_a-\alpha\,\Xi\,T_S+\theta\,\phi\,T_S+\alpha\,\Xi\,T_a\,T_S-\theta\,\phi\,T_a\,T_S+\Xi\,T_a\,\sigma_a-\Xi\,T_a\,T_S\,\sigma_a-\alpha\,\sigma_S+\alpha\,T_a\,\sigma_S-T_a\,\sigma_a\,\sigma_S}{-\alpha+\alpha\,T_a-T_a\,\sigma_a} \\ \Sigma & \frac{1}{\sigma_a} & \sigma_S \end{pmatrix}$$

$\gamma 1[A] - $ **DiagonalMatrix**$\left[\left\{\frac{1}{\sigma_a}, \sigma_S\right\}\right]$ // **Simplify** // **MatrixForm**

$$\begin{pmatrix} \frac{\alpha\,(-1+T_a)}{\sigma_a\,(\alpha+T_a\,(-\alpha+\sigma_a))} & \frac{\theta\,(-1+T_a)}{-\alpha+T_a\,(\alpha-\sigma_a)} \\ \frac{\phi\,T_a\,(-1+T_S)}{-\alpha+T_a\,(\alpha-\sigma_a)} & \frac{(-1+T_S)\,(-\alpha\,\Xi+\theta\,\phi+T_a\,(\alpha\,\Xi-\theta\,\phi-\Xi\,\sigma_a))}{-\alpha+T_a\,(\alpha-\sigma_a)} \end{pmatrix}$$

Clear[α, θ, ϕ, Ξ, ω];

$\gamma 0 = \Gamma\left[\omega,\ h_a\,\sigma_a + h_S\,\sigma_S,\ \{t_a, t_S\}.\begin{pmatrix} \sigma_a + \alpha\,(T_a - 1) & \theta\,(T_a - 1) \\ \phi\,(T_S - 1) & \sigma_S + \Xi\,(T_S - 1) \end{pmatrix}.\{h_a, h_S\}\right];$

$\gamma 1 = \gamma 0$ // **qΔ**[a, b, c]

$$\begin{pmatrix} \omega & s_b & s_c & s_S \\ s_b & -\alpha\,T_c+\alpha\,T_b\,T_c+\sigma_a & \alpha\,(-1+T_b)\,T_c & \theta\,(-1+T_b)\,T_c \\ s_c & \alpha\,(-1+T_c) & -\alpha+\alpha\,T_c+\sigma_a & \theta\,(-1+T_c) \\ s_S & \phi\,(-1+T_S) & \phi\,(-1+T_S) & -\Xi+\Xi\,T_S+\sigma_S \\ \Sigma & \sigma_a & \sigma_a & \sigma_S \end{pmatrix}$$

$\gamma 1[A] - $ **DiagonalMatrix**[$\{\sigma_a, \sigma_a, \sigma_S\}$] // **Simplify** // **MatrixForm**

$$\begin{pmatrix} \alpha\,(-1+T_b)\,T_c & \alpha\,(-1+T_b)\,T_c & \theta\,(-1+T_b)\,T_c \\ \alpha\,(-1+T_c) & \alpha\,(-1+T_c) & \theta\,(-1+T_c) \\ \phi\,(-1+T_S) & \phi\,(-1+T_S) & \Xi\,(-1+T_S) \end{pmatrix}$$