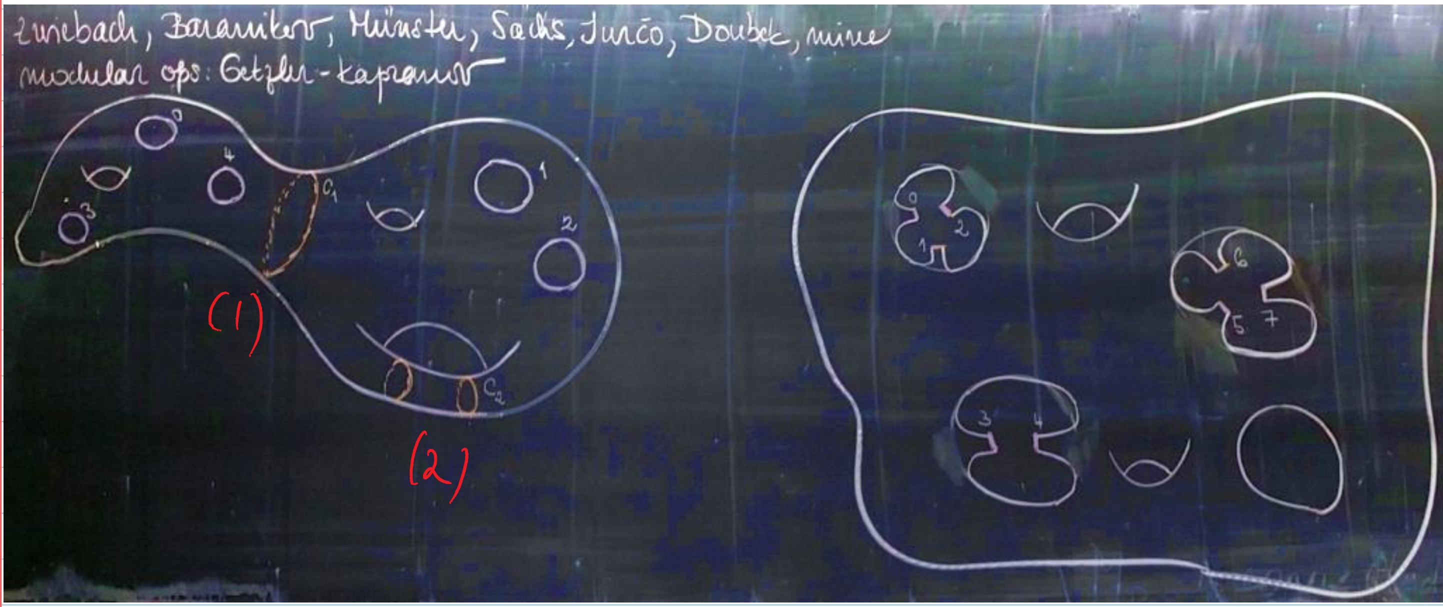
Markl: Algebraic structures of string field theory

October-22-13
7:59 AM



L_∞ Algebra: graded v.s. V,

$l_n : V^{\otimes n} \to V$, $n \geq 1$

axioms, . . .

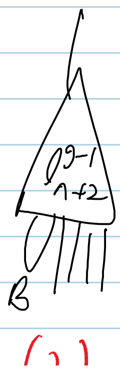
$\sum$  $= 0$

Zwiebach:

CSFT loop homotopy algebra, quantum L_∞ algebra

$l_n^g : V^{\otimes n} \to V$ $\quad B : V \otimes V \to V$

similar axioms,

$L_n^g = \sum_{\substack{i+j=n+1 \\ u+v=g}}$  $+ \frac{1}{2}$ 

$u+v=g$ (1)

$+$ cyclic symmetry:  is skew-symmetric

$\Longleftrightarrow$ $\{S,S\}+\frac{1}{2}\hbar\Delta=0$, $(\delta+\hbar)^2=0$ in some language

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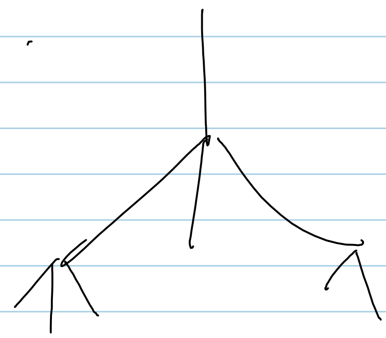
Operads: in d-g-v.s.

symmetric group

$P=\{P(n)\}_{n\geq 1}$ P_n acted on by Σ_n

Compositions using rooted trees.
or using "$\circ_i$".

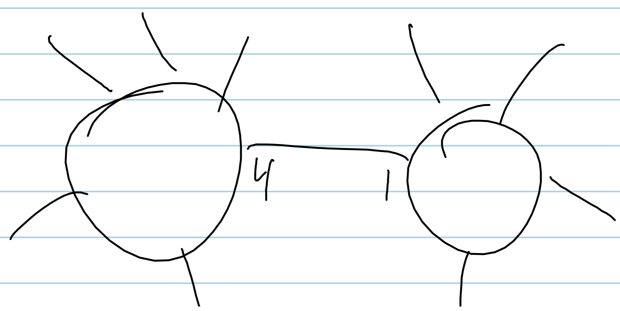


Example:
$\mathrm{End}_V(n):=\mathrm{Lin}(V^{\otimes n},V)$

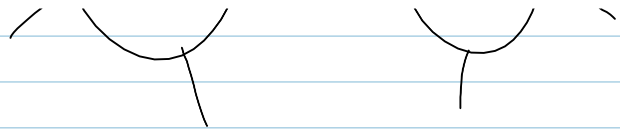
A P-algebra is an operad map $A: P\to\mathrm{End}_V$

Cyclic operads P_n has a Σ_{n+1} action

with compositions



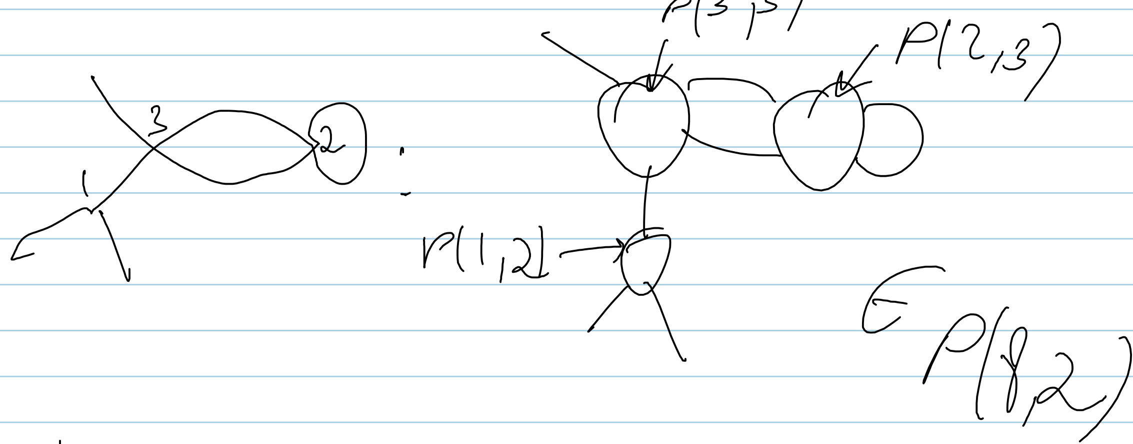
$P_1\circ_4 Q$



Modular Operads:

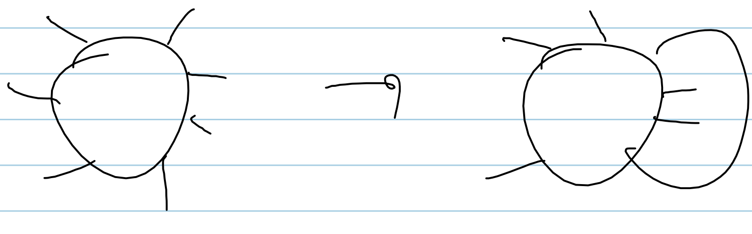
$$P = \{P(G,n)\}_{\substack{G \geq 0 \\ n \geq 1}}$$

Pasting schemes: arbitrary graphs with vertices marked by natural numbers



still have ${}_i\circ_j$ but now also

$$\xi_{ij} : P(G,n) \longrightarrow P(G+1, n-2)$$



Examples: Cyclic: V, non-deg form

$$End_V(n) := V^{\otimes n+1}$$

(also modular by ignoring G)

Modular: $Com = \{Com(n)\}$

$Com(n) = \mathbb{F}$, Σ_{n+1} trivial

$A : Com \to End_V$

is the same as Frobenius algebras.

Bar-construction A: associative algebra

$$B(A) = (\underset{\text{free}}{\mathbb{F}}(A^*), d)$$

d is the unique derivation s.t.

$$d|_{A^*} : A^* \to A^* \otimes A^*$$

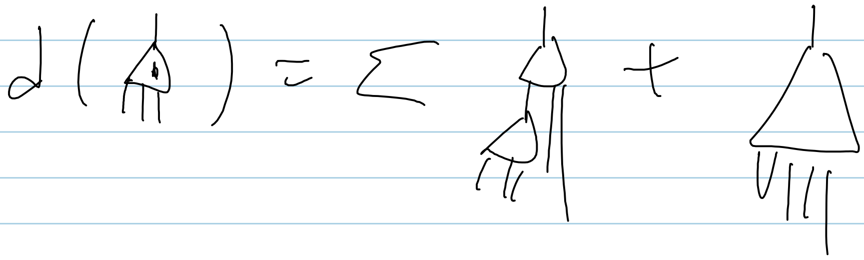
is dual to the product in A.

There is a similar construction for operads. An algebra over $B(P)$ is - - - -

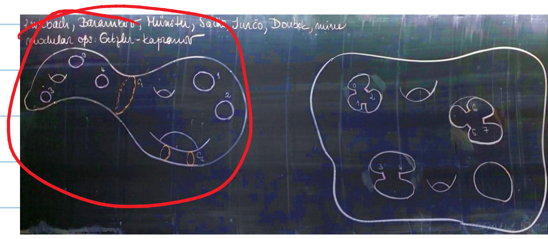
<u>Thm</u> $B(Com)$-algebras are the same as L_∞-algebra.

Bar for modular operads:

$$B(P) = (F(P^*), d)$$



The surfaces on the initial blackboard make a modular operad M.

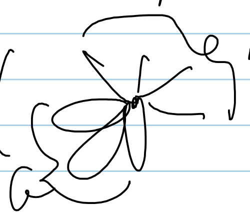


<u>Thm</u> $B(M)$-algebras are quantum L_∞ algebras.

$\exists$ functor modular ops $\longrightarrow$ cyclic ops

Mod: The left adjoint of that

Mod(P) "the modular envelope".

$$\mathrm{Mod}(\mathrm{Com})(G,n) = \mathrm{span}\Big($$  $$\Big)$$

There is an "open-string" version

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