Kotschick: Fundamental groups of Kahler manifolds and combinatorial group theory

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3:47 AM

"We don't want to be over-swiss, so we wait one minute".

Kähler groups: Γ is a Kähler group if it is π_1 of a closed Kähler manifold.

A Kähler group is finitely presented.

There is no characterization in sight.

<u>Def</u> Γ is a projective group if it is π_1 of a smooth projective variety/$\mathbb{C}$.

$$\underbrace{\mathcal{P}}_{\text{projective groups}} \underbrace{\subseteq}_{\text{strictness unknown}} \underbrace{\mathcal{K}}_{\text{Kähler groups}} \subsetneq \mathcal{F} \quad \text{finitely presented.}$$

<u>Examples</u> 0. Every finite group is Kähler, in fact projective.

1. A finitely generated Abelian group is Kähler/projective iff rank is even.

2. Free groups are not Kähler

[Abelianizations must be of even rank]
but free groups have finite index subgroup of odd ranks.

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negative results

Thm (Gromov): Kähler groups have finitely many ends.

Thm. (Carlson-Toledo) If Γ is a lattice (co-compact) in $Iso(\mathbb{H}^n)$ $n \geq 3$, then Γ is not Kähler.

both thms ~ 1989

The Higman 4-group is not Kähler.

positive results

Toledo (1990): $\exists$ Kähler group of every cohomological dimension ≥ 5.

Toledo (1993): $\exists$ non-residually finite Kähler groups (in fact projective).

M. Kapovich 1998: $\exists$ non-coherent Kähler groups [have finitely generated subgroups that are not finitely presented]

Thm 1 Thompson's group F is not Kähler (Napier–Ramachandra ~2005)

Thm 2 A group of deficiency 2 is Kähler iff it is $\pi_1^{orb}(C_g)$

Deficiency := $\max\limits_{\text{over all}} \{\#\text{gens} - \#\text{rels}\}$

presentations

Thm 3 An infinite 3-manifold group is Kähler iff it is $\pi_1(C_g)$.

not necessarily closed.

The argument for all 3 Theorems:

X: cmpct Kähler manifold, connected

$$alb_X : X \longrightarrow H^0(X, \Omega^1)^* / H_1(X, \mathbb{Z})$$

"The Albanese map"

by integrating paths from a fixed basepoint.

$f: X \to C$ is a holomorphic surjective map with compact fibers to a curve. C

Then

$$1 \longrightarrow K \longrightarrow \pi_1(X) \longrightarrow \pi_1^{orb}(C) \to 1$$

40 mins.