

"We don't want to be over-swiss, so we wait one minute".

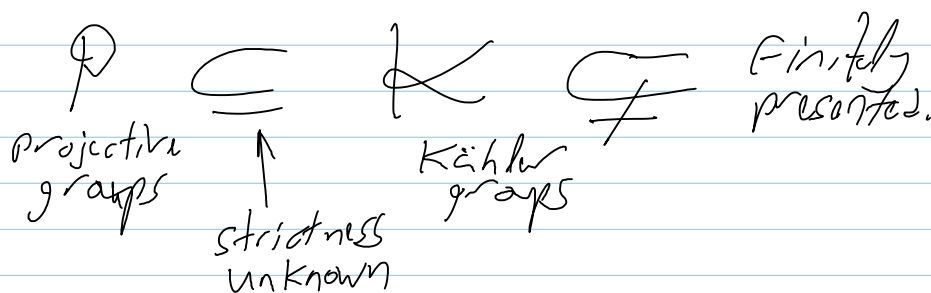
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Kähler groups:  $\Gamma$  is a Kähler group if it is  $\pi_1$  of a closed Kähler manifold.

A Kähler group is finitely presented.

There is no characterization in sight.

Def  $\Gamma$  is a projective group if it is  $\pi_1$  of a smooth projective variety/ $\mathbb{C}$ .



Examples 0. Every finite group is Kähler, in fact projective.

1. A finitely generated Abelian group is Kähler/projective iff rank is even.

2. Free groups are not Kähler

[Abelianizations must be of even rank]  
but free groups have finite index subgroup  
of odd ranks.

Negative results

Thm (Gromov): Kähler groups have finitely many ends.

Thm (Carlson-Toledo)

IF  $\Gamma$  is a lattice (co-compact) in

$\text{Iso}(\mathbb{H}^n)$   $n \geq 3$ , then  $\Gamma$  is not Kähler.

both thms  $\sim 1989$

The Higman 4-group is not Kähler.

Positive results

Toledo (1990):  $\exists$  Kähler group of every cohomological dimension  $\geq 5$ .

Toledo (1993):  $\exists$  non-residually finite Kähler groups (in fact projective).

M. Kapovich 1998:  $\exists$  non-coherent Kähler groups [have finitely generated subgroups that are not finitely presented]

Thm 1 Thompson's group  $F$  is not Kähler (Napier-Ramachandra  $\sim 2005$ )

Thm 2 A group of deficiency 2 is Kähler iff it is  $\Pi_1^{\text{orb}}(C_g)$

Deficiency :=  $\max_{\text{over all}} \{ \# \text{gens} - \# \text{rels} \}$

presentations

Thm 3 An infinite 3-manifold group is  
Kähler iff it is  $\pi_1(C_g)$

↑  
not necessarily  
closed.

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The argument for all 3 Theorems:

$X$ : compact Kähler manifold, connected

$$\text{alb}_X : X \longrightarrow H^0(X, \mathcal{O}^1)^* / H_1(X, \mathbb{Z})$$

"The Albanese map"

by integrating paths from a fixed  
basepoint.

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$f : X \rightarrow C$  is a holomorphic surjective  
map with compact fibers to a curve  $C$

Then

$$1 \rightarrow K \rightarrow \pi_1(X) \rightarrow \pi_1^{\text{orb}}(C) \rightarrow 1$$

40mins.