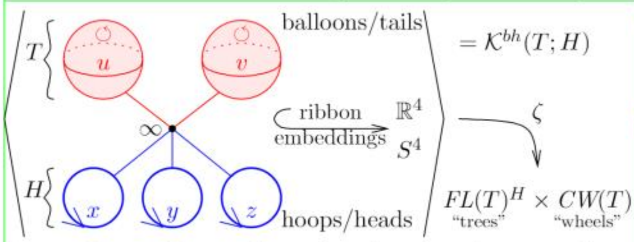
Geneva 131024 Handout as of 131022

October-22-13
2:05 PM

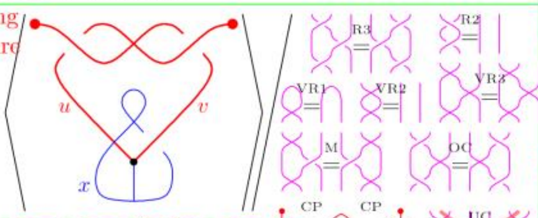
Finite Type Invariants of Ribbon Knotted Balloons and Hoops

Abstract. On my September 17 Geneva talk (ω/sep) I described a certain trees-and-wheels-valued invariant ζ of ribbon knotted loops and 2-spheres in 4-space, and my October 8 Geneva talk (ω/oct) describes its reduction to the Alexander polynomial. Today I will explain how that same invariant arises completely naturally within the theory of finite type invariants of ribbon knotted loops and 2-spheres in 4-space.

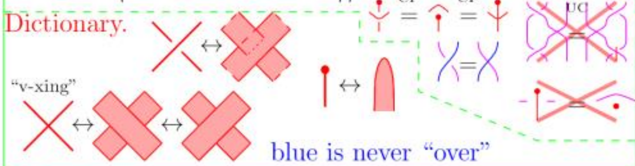


My goal is to tell you why such an invariant is expected, yet not to derive the formulas.

Disturbing Conjecture



Dictionary.



Expansions the semi-virtual $:=$ ... i.e. ... or ...

Let $\mathcal{I}^n := \langle$pictures with $\geq n$ semi-virts$\rangle \subset \mathcal{K}^{bh}$. We seek an "expansion"

$$Z\colon \mathcal{K}^{bh} \to \operatorname{gr}\mathcal{K}^{bh} = \widehat{\bigoplus}\mathcal{I}^n/\mathcal{I}^{n+1} =: \mathcal{A}^{bh}$$

satisfying "property U": if $\gamma \in \mathcal{I}^n$, then

$$Z(\gamma) = (0,\dots,0,\gamma/\mathcal{I}^{n+1},*,*,\dots).$$

Why? • Just because, and this is vastly more general.

• $(\mathcal{K}^{bh}/\mathcal{I}^{n+1})^\star$ is "finite-type/polynomial invariants".

• The Taylor example: Take $\mathcal{K} = C^\infty(\mathbb{R}^n)$, $\mathcal{I} = \{f \in \mathcal{K}\colon f(0)=0\}$. Then $\mathcal{I}^n = \{f\colon f \text{ vanishes like } |x|^n\}$ so $\mathcal{I}^n/\mathcal{I}^{n+1}$ is homogeneous polynomials of degree n and Z is a "Taylor expansion"! (So Taylor expansions are vastly more general than you'd think).

Plan. We'll construct a graded $\tilde{\mathcal{A}}^{bh}$, a surjective graded $\pi\colon \tilde{\mathcal{A}}^{bh} \to \mathcal{A}^{bh}$, ~~and a filtered~~ $\tilde{Z}\colon \mathcal{K}^{bh} \to \mathcal{A}^{bh}$ so that $\pi /\!\!/ \operatorname{gr}\tilde{Z} = Id$. It follows that • π is an isomorphism. • $Z := \tilde{Z} /\!\!/ \pi$ is an expansion.

$$\begin{tikzcd} & \tilde{\mathcal{A}}^{bh} \arrow[d, shift left, "\pi"'] \\ \mathcal{K}^{bh} \arrow[ur, "\tilde{Z}"] \arrow[r, "Z"'] & \mathcal{A}^{bh} \arrow[u, shift left, "\operatorname{gr}\tilde{Z}"'] \end{tikzcd}$$

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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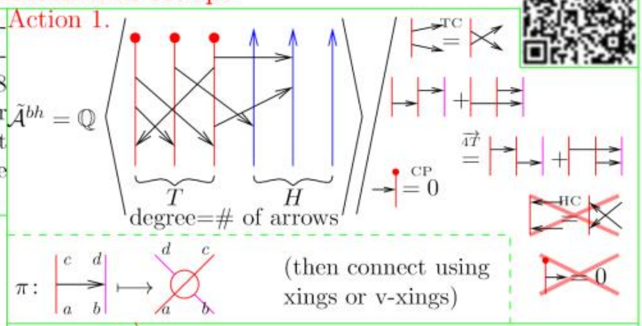
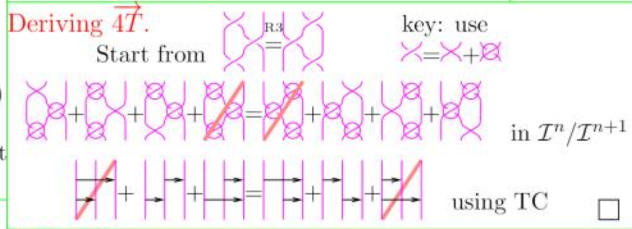
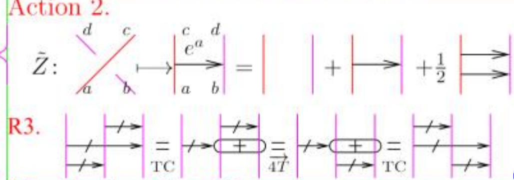
Action 1.



Deriving $\overrightarrow{4T}$. Start from ... key: use ...

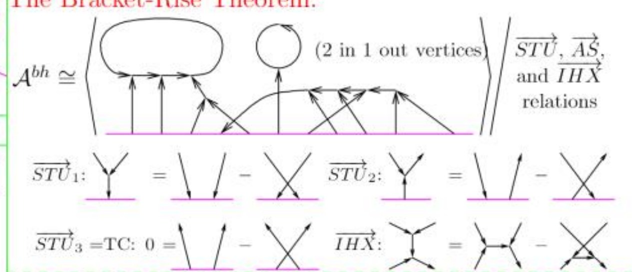


Action 2.



Ex: Z is an expansion

The Bracket-Rise Theorem.

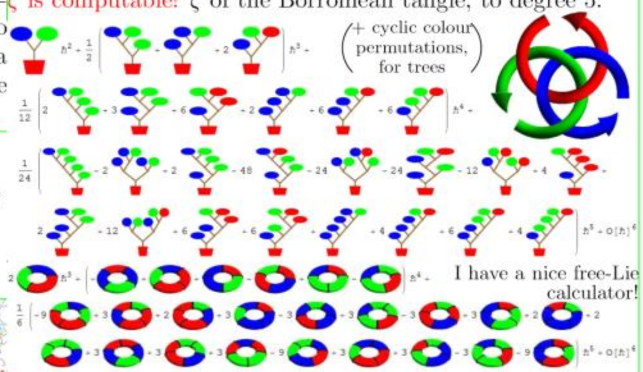


Proof.

Corollaries. (1) Related to Lie algebras! (2) Only trees and wheels persist.

Theorem. $\mathcal{A}^{bh}$ is a bi-algebra. The space of its primitives is $FL(T)^H \times CW(T)$, and $\zeta = \log Z$.

ζ is computable! ζ of the Borromean tangle, to degree 5:



I have a nice free-Lie calculator!

(meaning, if $\deg K = n$, $\tilde{Z}(\Pi(K)) = \Pi(K) +$ higher order terms)