The cocycle property for div in algebraic notation

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5:47 PM

Compare w/ 2013-03

$$\mathrm{div}_u\alpha = C(\mathrm{ad}_u^\nu \alpha)$$

Of course, the right letter to use here is \sampi, not ν.

$$C(\alpha /\!/ \mathrm{ad}_u^\nu /\!/ \mathrm{ad}_u^\beta) = C(\alpha /\!/ \mathrm{ad}_u^\nu) /\!/ \mathrm{ad}_u^\beta \qquad (1)$$

$$C(\alpha /\!/ \mathrm{ad}_u^{[\nu,\beta]}) = C([\mathrm{ad}_u^\nu \alpha, \beta]) \qquad (2)$$

$$C(\alpha /\!/ \mathrm{ad}_u^{\beta /\!/ \mathrm{ad}_u^\nu}) = C(\beta /\!/ \mathrm{ad}_u^{\alpha /\!/ \mathrm{ad}_u^\nu}) \qquad (3)$$

$$\mathrm{ad}_u^\beta /\!/ \mathrm{ad}_u^\nu = \mathrm{ad}_u^\nu /\!/ \mathrm{ad}_u^\beta \pm \mathrm{ad}_u^{[\nu,\beta]} \pm \mathrm{ad}_u^{\beta /\!/ \mathrm{ad}_u^\nu} \qquad (4)$$

$$\underbrace{(\mathrm{div}_u\,\alpha) /\!/ \mathrm{ad}_u^\beta}_{A} - \underbrace{(\mathrm{div}_u\,\beta) /\!/ \mathrm{ad}_u^\alpha}_{B} = \underbrace{\mathrm{div}_u[\alpha,\beta]}_{C} + \underbrace{\mathrm{div}_u(\alpha /\!/ \mathrm{ad}_u^\beta)}_{D} - \underbrace{\mathrm{div}_u(\beta /\!/ \mathrm{ad}_u^\alpha)}_{E}.$$

$$A = C(\mathrm{ad}_u^\nu \alpha) /\!/ \mathrm{ad}_u^\beta = C(\alpha /\!/ \mathrm{ad}_u^\nu /\!/ \mathrm{ad}_u^\beta)$$ using (1)

$B =$

$$C = C(\mathrm{ad}_u^\nu[\alpha,\beta]) = C([\mathrm{ad}_u^\nu \alpha, \beta] + [\alpha, \mathrm{ad}_u^\nu \beta])$$

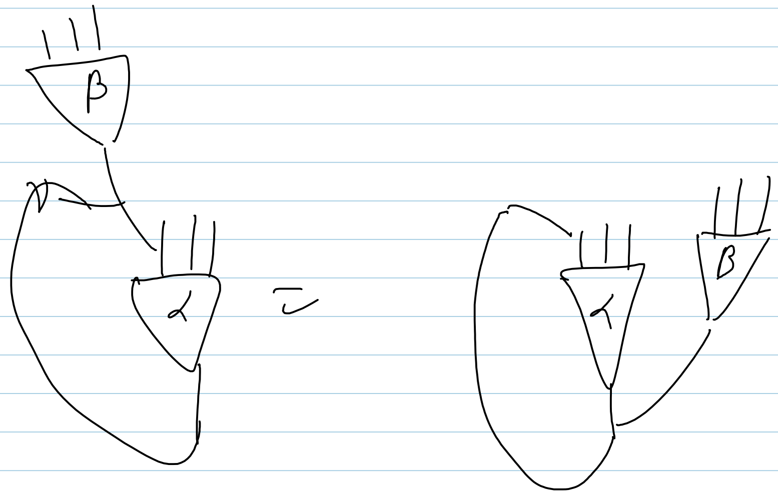
$$D = C(\alpha /\!/ \mathrm{ad}_u^\beta /\!/ \mathrm{ad}_u^\nu) = C(\alpha /\!/ \mathrm{ad}_u^\nu /\!/ \mathrm{ad}_u^\beta) \pm C(\alpha /\!/ \mathrm{ad}_u^{[\nu,\beta]}) \pm C(\alpha /\!/ \mathrm{ad}_u^{\beta /\!/ \mathrm{ad}_u \nu})$$ using (4)

$E =$

This does lead to a proof and all necessary identities are here. It is not clear if this is better than the pictorial proof:

$$D-A = \pm \underbrace{C(\alpha // ad_u^{[\nu,\beta]})}_{\text{using (2), cancels half of } C} \pm \underbrace{C(\alpha // ad_u^{\beta // ad_u \nu})}_{\text{using (3), cancels against the remaining part of } E-B.}$$

$$E-B = \pm \begin{pmatrix} \text{2nd half} \\ \text{of } C \end{pmatrix} \pm C(\beta // ad_u^{\alpha // ad_u \nu})$$



Pf of (2)

Recycling:

$$ad_u^\alpha // ad_u^\beta - ad_u^\beta // ad_u^\alpha = \pm ad_u^{[\alpha,\beta]} \pm \ldots$$