Jt property - proof

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$$J_u(\gamma) = \int_0^1 ds\, \gamma // RC_u^{s\gamma} // div_u // C_u^{-s\gamma}$$

$$\frac{d}{ds} J_u(s\gamma) = \gamma // RC_u^{\gamma} // div_u // C_u^{-\gamma}$$

With $\gamma_w = \gamma // tm_w^{uv}$, Property t_1

$$\underbrace{J_w(\gamma_w) // RC^{\gamma_w}}_{L} \overset{?}{=} \left.\begin{array}{l} J_u(\gamma) // tm_w^{uv} // RC_w^{\gamma_w} \quad \} R_1 \\ + J_v(\gamma // RC_u^{\gamma}) // RC_v^{\gamma // RC_u^{\gamma}} // tm_w^{uv} \quad \} R_2 \end{array}\right\} R$$

Note. $R_2 = J_v(-) // C_u^{-\gamma} // t_w^{uv} // RC_w^{\gamma_w}$ [using RC eqn †]

Replace γ by $s\gamma$ everywhere and differentiate:

$$\frac{d}{ds} L(s) = \gamma_w // RC_w^{\gamma_w} // div_w + J_w(\gamma_w) // RC_w^{\gamma_w} // ad_w^{\gamma_w // RC_w^{\gamma_w}}$$

$$= \gamma // t_w^{uv} // RC_w^{\gamma // t_w^{uv}} // div_w +$$

$$= \gamma //$$

Property t_1 is equiv. to [post-compose $C_w^{-\gamma_w}$] w/

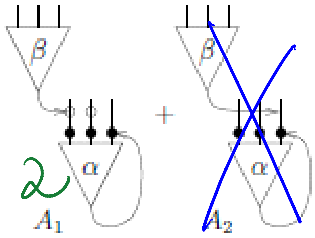
$$J_w(\gamma_w) = J_u(\gamma) // t_w^{uv} + J_v(\gamma // RC_u^{\gamma}) // C_u^{-\gamma} // t_w^{uv}$$ } Put on cheat sheet ?!

There ought to be a u-v cocycle eq'n for div:

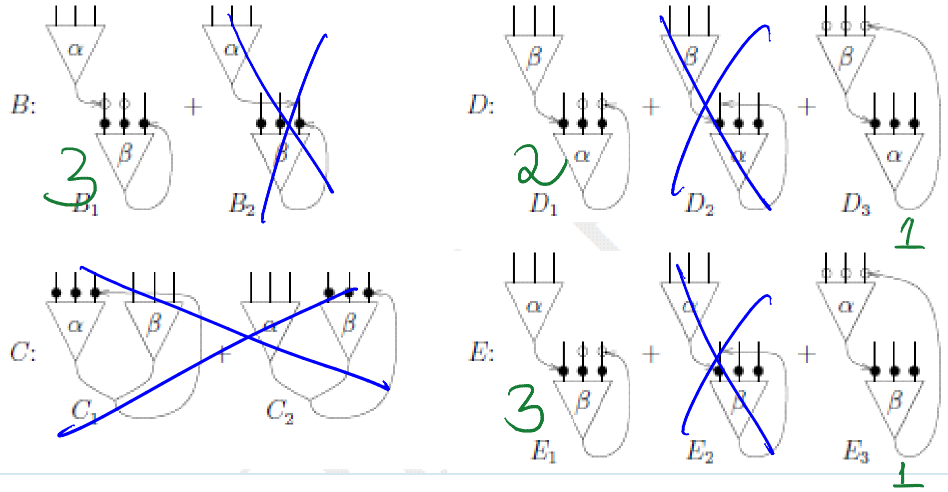
$$\left.\begin{array}{l} \underbrace{\alpha // div_u}_{A} // ad_v^{\beta} - \underbrace{\beta // div_v}_{B} // ad_u^{\alpha} \overset{?}{=} \\ \alpha // \underbrace{ad_v^{\beta}}_{D} // div_u - \beta // \underbrace{ad_u^{\alpha}}_{E} // div_v \end{array}\right\}$$ Put on cheat sheet ?!

Proof ...

<u>Proof</u> Pretend α has a u-head & β has a v-head



cancellations in green



$$\underbrace{(\operatorname{div}_u \alpha) /\!\!/ \operatorname{ad}_u^\beta}_{A} - \underbrace{(\operatorname{div}_u \beta) /\!\!/ \operatorname{ad}_u^\alpha}_{B} = \underbrace{\operatorname{div}_u[\alpha,\beta]}_{C} + \underbrace{\operatorname{div}_u(\alpha /\!\!/ \operatorname{ad}_u^\beta)}_{D} - \underbrace{\operatorname{div}_u(\beta /\!\!/ \operatorname{ad}_u^\alpha)}_{E}$$

$\square$