

$$\underbrace{J_u(\text{bch}(\alpha, \beta))}_{\text{LHS}} \stackrel{?}{=} \underbrace{J_u(\alpha) + J_u(\beta // RC_u^\alpha)}_{\text{RHS}} // C_u^{-\alpha}$$

silly! It is enough to prove that

$$J_u(\text{bch}(\alpha, s\beta)) = J_u(\alpha) + J_u(s\beta // RC_u^\alpha) // C_u^{-\alpha}$$

for every s . w/ $\gamma = \text{bch}(\alpha, s\beta)$

$$\frac{d}{ds} \text{LHS} = \beta // RC_u^\gamma // \text{div}_u // C_u^{-\gamma}$$

$$\frac{d}{ds} \text{RHS} = \beta // RC_u^\alpha // \underbrace{\frac{1 - e^{-\text{ad } \beta // RC_u^\alpha}}{\text{ad } \cdot}}_{\substack{\text{commutes \&} \\ \text{disappears}}} // RC_u^{\beta // RC_u^\alpha} // \text{div}_u // C_u^{-\beta // RC_u^\alpha} // C_u^{-\alpha}$$

= same thing. □

Question does $\gamma_s := \text{bch}(\alpha, s\beta)$ commute

with $\frac{d}{ds} \gamma_s$?

$$\frac{d}{ds} \gamma_s = \beta // \frac{\text{ad } \gamma_s}{1 - e^{-\text{ad } \gamma_s}} \dots \text{No.}$$