Cheat Sheet J on April 12, 2013, 4:04PM

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Cheat Sheet J

http://drorbn.net/AcademicPensieve/2013-04/
initiated 18/3/13; continues 2013-03; modified 12/4/13, 4:04pm

With alphabet T and with $u,v,w \in T$, $\alpha,\beta,\gamma \in FL(T)$, $D \in \mathfrak{tder}(T)$, $g,h \in \exp(\mathfrak{tder}(T)) = \mathrm{TAut}(T)$. Checkmarks (✓) as in `CheatSheetJ-Verification.nb`.

verify bw form

1. The definition of J: $\quad J_u(\gamma) := \int_0^1 ds\, \mathrm{div}_u(\gamma \,/\!\!/\, RC_u^{s\gamma}) \,/\!\!/\, C_u^{-s\gamma}$

2. ✓ The t equation (desired; obvious splitting seems to fail):
$$J_w(\gamma \,/\!\!/\, tm_w^{uv}) \,/\!\!/\, RC_w^{\gamma /\!\!/ tm_w^{uv}} = J_u(\gamma) \,/\!\!/\, tm_w^{uv} \,/\!\!/\, RC_w^{\gamma /\!\!/ tm_w^{uv}} + J_v(\gamma \,/\!\!/\, RC_u^\gamma) \,/\!\!/\, RC_v^{\gamma /\!\!/ RC_u^\gamma} \,/\!\!/\, tm_w^{uv}$$ A ✓

3. ✓ The h equation (desired): $\quad J_u(\mathrm{bch}(\alpha,\beta)) = J_u(\alpha) + J_u(\beta \,/\!\!/\, RC_u^\alpha) \,/\!\!/\, C_u^{-\alpha}$

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4. ✓ The meaning(s) of RC: $\quad C_u^\gamma \,/\!\!/\, RC_u^{-\gamma} = Id, \qquad C_u^{\gamma /\!\!/ RC_u^\gamma} = RC_u^\gamma$

5. RC equation t: $\quad tm_w^{uv} \,/\!\!/\, RC_w^{\gamma /\!\!/ tm_w^{uv}} = RC_u^\gamma \,/\!\!/\, RC_v^{\gamma /\!\!/ RC_u^\gamma} \,/\!\!/\, tm_w^{uv}$

6. RC equation h: $\quad RC_u^{\mathrm{bch}(\alpha,\beta)} = RC_u^\alpha \,/\!\!/\, RC_u^{\beta /\!\!/ RC_u^\alpha}$

7. RCC equation div: $\quad \mathrm{div}_u(\alpha \,/\!\!/\, RC_u^\gamma) \,/\!\!/\, C_u^\gamma = ?$

8. CRC equation div: $\quad \mathrm{div}_u(\alpha \,/\!\!/\, C_u^\gamma) \,/\!\!/\, RC_u^\gamma = ?$

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9. div property ~~t~~ w ✓ $\quad \mathrm{div}_w(\gamma \,/\!\!/\, tm_w^{uv}) = (\mathrm{div}_u(\gamma) + \mathrm{div}_v(\gamma)) \,/\!\!/\, tm_w^{uv}$ B ✓

10. ✓ div property h — the "cocycle condition": with $\mathrm{ad}_u\{\gamma\} := \mathrm{der}(u \to [\gamma, u])$,
$$(\mathrm{div}_u\,\alpha) \,/\!\!/\, \mathrm{ad}_u\{\beta\} - (\mathrm{div}_u\,\beta) \,/\!\!/\, \mathrm{ad}_u\{\alpha\} = \mathrm{div}_u\left([\alpha,\beta] + \alpha \,/\!\!/\, \mathrm{ad}_u\{\beta\} - \beta \,/\!\!/\, \mathrm{ad}_u\{\alpha\}\right)$$

~~11. div of bch: $\quad \mathrm{div}_u(\mathrm{bch}(\alpha,\beta)) = ?$~~ remove

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12. The definition of JA: $\quad JA_u(\gamma) := J_u(\gamma) \,/\!\!/\, RC_u^\gamma$

13. The ODE for JA: with $\gamma_s = \gamma \,/\!\!/\, RC_u^{s\gamma}$,
$$JA(0) = 0, \qquad \frac{dJA(s)}{ds} = JA(s) \,/\!\!/\, \mathrm{ad}_u\{\gamma_s\} + \mathrm{div}_u\,\gamma_s, \qquad JA(1) = JA_u(\gamma)$$

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14. The relation with $\mathfrak{tder}$: $\quad e^{\mathrm{ad}_u\{\gamma\}} = C_u^{?}$ and $C_u^\gamma = e^{\mathrm{ad}_u\{?\}}$

15. The definition of j (following A-T): $\quad j(e^D) = \int_0^1 ds\, e^{sD}(\mathrm{div}\, D) = \frac{e^D - 1}{D}(\mathrm{div}\, D)$

16. j's cocycle property: $\quad j(gh) = j(g) + g \cdot j(h)$

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17. The differential of exp: $\quad \delta e^\gamma = e^\gamma \cdot \left(\delta\gamma \,/\!\!/\, \frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right) = \left(\delta\gamma \,/\!\!/\, \frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma}\right) \cdot e^\gamma$

18. ✓ The differential of $\gamma = \mathrm{bch}(\alpha,\beta)$:
$$\delta\gamma \,/\!\!/\, \frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma} = \left(\delta\alpha \,/\!\!/\, \frac{1-e^{-\mathrm{ad}\,\alpha}}{\mathrm{ad}\,\alpha} \,/\!\!/\, e^{-\mathrm{ad}\,\beta}\right) + \left(\delta\beta \,/\!\!/\, \frac{1-e^{-\mathrm{ad}\,\beta}}{\mathrm{ad}\,\beta}\right)$$

19. ✓ The differential of C: $\quad \delta C_u^\gamma = \mathrm{ad}_u\left\{\delta\gamma \,/\!\!/\, \frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma} \,/\!\!/\, RC_u^{-\gamma}\right\} \,/\!\!/\, C_u^\gamma$

20. ✓ The differential of RC: $\quad \delta RC_u^\gamma = RC_u^\gamma \,/\!\!/\, \mathrm{ad}_u\left\{\delta\gamma \,/\!\!/\, \frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma} \,/\!\!/\, RC_u^\gamma\right\}$

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21. ✓ The differential of J: $\quad \delta J_u(\gamma) = \delta\gamma \,/\!\!/\, \frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma} \,/\!\!/\, RC_u^\gamma \,/\!\!/\, \mathrm{div}_u \,/\!\!/\, C_u^{-\gamma}$

A: $J_w(\gamma_w) = J_u(\gamma) \,/\!\!/\, t_w^{uv} + J_v(\gamma \,/\!\!/\, RC_u^\gamma) \,/\!\!/\, C_u^{-\gamma} \,/\!\!/\, t_w^{uv}$ ✓

B: $\alpha /\!/ \mathrm{div}_u /\!/ \mathrm{ad}_v^\beta - \beta /\!/ \mathrm{div}_v /\!/ \mathrm{ad}_u^\alpha =$

$\alpha /\!/ \mathrm{ad}_v^\beta /\!/ \mathrm{div}_u - \beta /\!/ \mathrm{ad}_u^\alpha /\!/ \mathrm{div}_v$ ✓

C_uC_v: $\quad C_u^{\gamma /\!/ RC_v^{-\gamma}} /\!/ C_v^\gamma = C_v^{\gamma /\!/ RC_u^{-\gamma}} /\!/ C_u^\gamma$

RC_uRC_v: $\quad RC_u^\gamma /\!/ RC_v^{\gamma /\!/ RC_u^\gamma} = RC_v^\gamma /\!/ RC_u^{\gamma /\!/ RC_v^\gamma}$

confirm & add. ✓