

Smaller font? Narrower margins?

Cheat Sheet \$\beta\$

<http://drorbn.net/AcademicPensieve/2013-03/>

initiated 24/3/13; continues 2013-03; modified 8/4/13, 11:26am; completed ?

The original \$\beta\$-calculus: With \$\epsilon := 1 + \alpha\$, \$\langle \alpha \rangle := \sum_v \alpha_v\$, and \$\langle \gamma \rangle := \sum_{v \neq u} \gamma_v\$,

$$\frac{\omega_1}{T_1} \left| \begin{array}{c|c} H_1 & \\ \hline A_1 & \end{array} \right| * \frac{\omega_2}{T_2} \left| \begin{array}{c|c} H_2 & \\ \hline A_2 & \end{array} \right| =_{\beta} \frac{\omega_1 \omega_2}{T_1 T_2} \left| \begin{array}{c|cc} H_1 & H_2 & \\ \hline A_1 & 0 & \\ & 0 & A_2 \end{array} \right|$$

$$\frac{\omega}{u} \left| \begin{array}{c|c} H & \\ \hline \alpha & \\ & \beta \\ & \gamma \end{array} \right| \xrightarrow{tm_{uv}^{uv}} \frac{\omega}{w} \left| \begin{array}{c|c} H & \\ \hline (\alpha + \beta) // \left(\begin{array}{c} u,v \\ \rightarrow w \end{array} \right) & \\ & \gamma \end{array} \right|$$

$$R_{ux}^{\pm} = \frac{1}{\beta} \left| \begin{array}{c|c} x & \\ \hline u & t_u^{\pm 1} - 1 \end{array} \right|$$

$$\frac{\omega}{T} \left| \begin{array}{ccc|c} x & y & H & \\ \hline \alpha & \beta & \gamma & \end{array} \right| \xrightarrow{hm_z^{xy}} \frac{\omega}{T} \left| \begin{array}{c|c} z & H \\ \hline \alpha + \beta + \langle \alpha \rangle \beta & \gamma \end{array} \right|$$

$$\frac{\omega}{u} \left| \begin{array}{ccc|c} x & H & & \\ \hline \alpha & \beta & & \\ & \gamma & \delta & \end{array} \right| \xrightarrow{sw_{th}^{ux}} \frac{\omega \epsilon}{T} \left| \begin{array}{c|cc} x & & H \\ \hline \alpha(1 + \langle \gamma \rangle / \epsilon) & \beta(1 + \langle \gamma \rangle / \epsilon) & \\ & \gamma / \epsilon & \delta - \gamma \beta / \epsilon \end{array} \right|$$

Constraints. • Column sums are monomials minus 1.

\$\beta\$-better calculus:

$$\frac{\omega_1}{T_1} \left| \begin{array}{c|c} H_1 & \\ \hline A_1 & \end{array} \right| * \frac{\omega_2}{T_2} \left| \begin{array}{c|c} H_2 & \\ \hline A_2 & \end{array} \right| =_{\beta_b} \frac{\omega_1 \omega_2}{T_1 T_2} \left| \begin{array}{c|cc} H_1 & H_2 & \\ \hline \omega_2 A_1 & 0 & \\ & 0 & \omega_1 A_2 \end{array} \right|$$

$$\frac{\omega}{u} \left| \begin{array}{c|c} H & \\ \hline \alpha & \\ & \beta \\ & \gamma \\ & \sigma \end{array} \right| \xrightarrow{tm_{uv}^{uv}} \frac{\omega}{w} \left| \begin{array}{c|c} H & \\ \hline (\alpha + \beta) // \left(\begin{array}{c} u,v \\ \rightarrow w \end{array} \right) & \\ & \gamma \\ & \sigma \end{array} \right|$$

$$R_{ux}^{\pm} = \frac{1}{\beta_b} \left| \begin{array}{c|c} x & \\ \hline u & t_u^{\pm 1} - 1 \end{array} \right|$$

$$\frac{\omega}{T} \left| \begin{array}{ccc|c} x & y & H & \\ \hline \alpha & \beta & \gamma & \end{array} \right| \xrightarrow{hm_z^{xy}} \frac{\omega}{T} \left| \begin{array}{c|c} z & H \\ \hline \alpha + \sigma_x \beta & \gamma \end{array} \right|$$

$$\frac{\omega}{u} \left| \begin{array}{ccc|c} x & H & & \\ \hline \alpha & \beta & & \\ & \gamma & \delta & \\ & \sigma_x & \sigma & \end{array} \right| \xrightarrow{sw_{th}^{ux}} \frac{\omega + \alpha}{u} \left| \begin{array}{c|cc} x & & H \\ \hline \sigma_x \alpha & \sigma_x \beta & \\ & \gamma & \delta + \frac{\alpha \delta - \gamma \beta}{\omega} = \frac{\omega + \alpha}{\omega} \delta - \frac{\gamma \beta}{\omega} \end{array} \right|$$

Constraints. • Sum of column \$x\$ is \$(\sigma_x - 1) \omega\$. • Likely, \$\omega^{k-1} | \Lambda^k A\$.

Common denominator

missing the \$\sigma\$ line

From 2012-05/A Higher Minors Experiment:

$$\frac{1}{\omega^2} \left| \begin{array}{cc|c} (\omega + \alpha) \delta_{11} - \gamma_1 \beta_1 & (\omega + \alpha) \delta_{12} - \gamma_1 \beta_2 & \\ \hline (\omega + \alpha) \delta_{21} - \gamma_2 \beta_1 & (\omega + \alpha) \delta_{22} - \gamma_2 \beta_2 & \end{array} \right| =$$

$\begin{matrix} < \beta_1 & \beta_2 \\ \gamma_1 & \delta_{11} & \delta_{12} \\ \gamma_2 & \delta_{21} & \delta_{22} \\ \text{starting } A \end{matrix}$

$$\frac{1}{\omega^2} \left[-\gamma_1 (\omega + \alpha) \left| \begin{array}{cc} \beta_1 & \beta_2 \\ \delta_{21} & \delta_{22} \end{array} \right| - \gamma_2 (\omega + \alpha) \left| \begin{array}{cc} \delta_{11} & \delta_{12} \\ \beta_1 & \beta_2 \end{array} \right| + (\omega + \alpha)^2 \left| \begin{array}{cc} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{array} \right| \right]$$

$$= \frac{\omega + \alpha}{\omega^2} \left[\omega \left| \begin{array}{cc} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{array} \right| + \left| \begin{array}{ccc} \alpha & \beta_1 & \beta_2 \\ \gamma_1 & \delta_{11} & \delta_{12} \\ \gamma_2 & \delta_{21} & \delta_{22} \end{array} \right| \right]$$

divisible by \$\omega\$
div by \$\omega^2\$

div by \$\omega^2\$

div by \$\omega + \alpha\$

larger

To do. • Consider a verification program. • Add \$dm\$ formulas. • Add Burau calculus.

* Add The conjugation relation

Add the MVA Formula:

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 $\beta$ MVA[L_Link] := Module[{ $\eta$ s,  $\omega$ ,  $\mu$ , M},
  { $\omega$ ,  $\mu$ } = List @@  $\beta$ Z[L];
   $\eta$ s = Rest[h# & /@ (First /@ Skeleton[L])];
  M = Outer[
    Coefficient[ $\mu - (\mu /. t_ \rightarrow 1 /. h_a_ \Rightarrow t_a h_a)$ , #1 * #2] &,
     $\eta$ s,  $\eta$ s /. h_a_ \Rightarrow t_a];
  Factor[ $\frac{\omega \text{Det}[M]}{1 - T_{\text{Skeleton}[L][[1,1]}}$ ] ] ]

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$\omega + \alpha$	x	H
u	$\sigma_x \alpha$	$\sigma_x \beta$
T	$\delta + \frac{\alpha\delta - \gamma\beta}{\omega}$	$\frac{(\omega + \alpha)\delta - \gamma\beta}{\omega}$
$-$	σ_x	σ

Aside: $\alpha + \gamma = \omega(\sigma_x - 1) \Rightarrow$
 $(\omega + \alpha)(\sigma_x - 1) = \alpha + \gamma + \alpha(\sigma_x - 1)$
 $= \sigma_x \alpha + \gamma$

$\gamma = \frac{(\omega + \alpha)\gamma - \gamma\alpha}{\omega}$ so this is a row operation!

$\omega \epsilon$	x	H
u	$\alpha(1 + \langle \gamma \rangle / \epsilon)$	$\beta(1 + \langle \gamma \rangle / \epsilon)$
T	γ / ϵ	$\delta - \gamma\beta / \epsilon$

$\frac{\gamma}{\epsilon} = \frac{\gamma}{1 + \alpha} = \gamma - \frac{\gamma\alpha}{\epsilon}$

