Cheat Sheet: Inflation

April-29-13
9:12 AM

KBH.

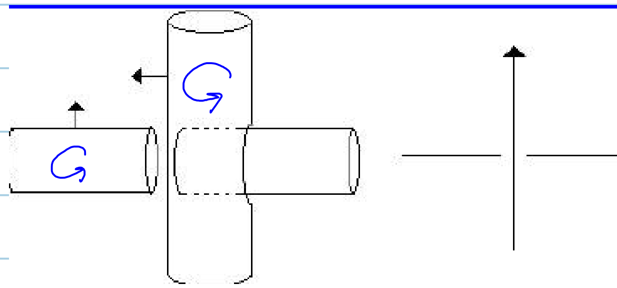
We can now define[7] a map δ_0, defined on v-knots and taking values in ribbon tori in $\mathbb{R}^4$: given (Σ, γ), embedd Σ arbitrarily in $\mathbb{R}^3_{xyz} \subset \mathbb{R}^4$. We say that a normal vector to Σ in $\mathbb{R}^4$ is "near unit" if its norm is between $1-\epsilon$ and $1+\epsilon$. The near-unit normal bundle of Σ has as fiber an annulus that can be identified with $[-\epsilon, \epsilon] \times S^1$ (first trivialize it using its positive-t-direction section), and hence the near-unit normal bundle of Σ defines an embedding of $\Sigma \times [-\epsilon, \epsilon] \times S^1$ into $\mathbb{R}^4$. On the other hand, γ is embedded in $\Sigma \times [-\epsilon, \epsilon]$ so $\gamma \times S^1$ is embedded in $\Sigma \times [-\epsilon, \epsilon] \times S^1$, and we can let $\delta_0(\Sigma, \gamma)$ be the composition

$$\gamma \times S^1 \hookrightarrow \Sigma \times [-\epsilon, \epsilon] \times S^1 \hookrightarrow \mathbb{R}^4,$$

which is a torus in $\mathbb{R}^4$. We leave it to the reader to verify that $\delta_0(\Sigma, \gamma)$ is ribbon, that it is independent of the choices made within its construction, that it is invariant under isotopies of γ and under tearing and puncturing, that it is also invariant under the "overcrossing commute" relation of Figure 3, and that it is equivalent to Satoh's tubing map.

The map δ_0 has straightforward generalizations to v-links, v-tangles, framed-v-links, v-knotted-graphs, etc.

Winter



$Tube(K) = -Tube(K)^*$

$-Tube(K) = Tube(-K)$

$Tube(K)^* = Tube(-K^\uparrow)$

$Tube(K) = Tube(-K^\uparrow)$

Satoh

For a surface-knot F, we use the notations $-F$ and F^* for the orientation-reversed and the mirror-imaged surface-knots of F respectively.