

Cheat Sheet J

http://drorbn.net/AcademicPensieve/2013-04/
initiated 18/3/13; continues 2013-03; continued 2013-12; modified 18/12/13, 12:14 Noon

With alphabet T and with $u, v, w \in T$, $\alpha, \beta, \gamma \in FL(T)$, $D \in \mathfrak{tder}(T)$, $g, h \in \exp(\mathfrak{tder}(T)) = \text{TAut}(T)$.
Checkmarks (\checkmark) as in `CheatSheetJ-Verification.nb`.

1. The definition of J :

$$J_u(\gamma) := \int_0^1 ds \operatorname{div}_u(\gamma \parallel RC_u^{s\gamma}) \parallel C_u^{-s\gamma}$$

2. \checkmark The J_{uv} equation:

$$J_u(\alpha) + J_v(\beta \parallel RC_u^\alpha) \parallel C_u^{-\alpha} = J_v(\beta) + J_u(\alpha \parallel RC_v^\beta) \parallel C_v^{-\beta}$$

3. \checkmark The t equation:

$$J_w(\gamma \parallel tm_w^{uv}) = (J_u(\gamma) + J_v(\gamma \parallel RC_u^\gamma) \parallel C_u^{-\gamma}) \parallel tm_w^{uv}$$

4. \checkmark The h equation:

$$J_u(\text{bch}(\alpha, \beta)) = J_u(\alpha) + J_u(\beta \parallel RC_u^\alpha) \parallel C_u^{-\alpha}$$

5. \checkmark The meaning(s) of RC :

$$C_u^\gamma \parallel RC_u^{-\gamma} = Id, \quad C_u^\gamma \parallel RC_u^\gamma = RC_u^\gamma$$

6. \checkmark $C_u C_v$ and $RC_u RC_v$:

$$C_u^\alpha \parallel RC_v^{-\beta} \parallel C_v^\beta = C_v^\beta \parallel RC_u^{-\alpha} \parallel C_u^\alpha, \quad RC_u^\alpha \parallel RC_v^\beta \parallel RC_u^\alpha = RC_v^\beta \parallel RC_u^\alpha \parallel RC_v^\beta$$

7. RC equation t :

$$tm_w^{uv} \parallel RC_w^\gamma \parallel tm_w^{uv} = RC_u^\gamma \parallel RC_v^\gamma \parallel RC_u^\gamma \parallel tm_w^{uv}$$

8. RC equation h :

$$RC_u^{\text{bch}(\alpha, \beta)} = RC_u^\alpha \parallel RC_u^\beta \parallel RC_u^\alpha$$

9. C -div- RC equations:

$$\operatorname{div}_u(\alpha \parallel RC_u^\gamma) \parallel C_u^\gamma = ? \quad \operatorname{div}_u(\alpha \parallel C_u^\gamma) \parallel RC_u^\gamma = ?$$

10. div property t :

$$\operatorname{div}_w(\gamma \parallel tm_w^{uv}) = (\operatorname{div}_u(\gamma) + \operatorname{div}_v(\gamma)) \parallel tm_w^{uv}$$

11. \checkmark div property uv : with $\operatorname{ad}_u^\gamma = \operatorname{ad}_u\{\gamma\} := \operatorname{der}(u \rightarrow [\gamma, u])$,

$$(\operatorname{div}_u \alpha) \parallel \operatorname{ad}_v^\beta - (\operatorname{div}_v \beta) \parallel \operatorname{ad}_u^\alpha = \operatorname{div}_u(\alpha \parallel \operatorname{ad}_v^\beta) - \operatorname{div}_v(\beta \parallel \operatorname{ad}_u^\alpha)$$

12. \checkmark div property uu : $(\operatorname{div}_u \alpha) \parallel \operatorname{ad}_u\{\beta\} - (\operatorname{div}_u \beta) \parallel \operatorname{ad}_u\{\alpha\} = \operatorname{div}_u([\alpha, \beta] + \alpha \parallel \operatorname{ad}_u\{\beta\} - \beta \parallel \operatorname{ad}_u\{\alpha\})$

13. The definition of JA :

$$JA_u(\gamma) := J_u(\gamma) \parallel RC_u^\gamma$$

14. The ODE for JA : with $\gamma_s = \gamma \parallel RC_u^{s\gamma}$,

$$JA(0) = 0, \quad \frac{dJA(s)}{ds} = JA(s) \parallel \operatorname{ad}_u\{\gamma_s\} + \operatorname{div}_u \gamma_s, \quad JA(1) = JA_u(\gamma)$$

15. The relation with \mathfrak{tder} : (there should be a many-variable version too) $e^{\operatorname{ad}_u\{\gamma\}} = C_u^\gamma$ and $C_u^\gamma = e^{\operatorname{ad}_u\{\gamma\}}$

16. The definition of j (following A-T):

$$j(e^D) = \int_0^1 ds e^{sD} (\operatorname{div} D) = \frac{e^D - 1}{D} (\operatorname{div} D)$$

17. j 's cocycle property:

$$j(gh) = j(g) + g \cdot j(h)$$

18. The differential of \exp :

$$\delta e^\gamma = e^\gamma \cdot \left(\delta\gamma \parallel \frac{1 - e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma} \right) = \left(\delta\gamma \parallel \frac{e^{\operatorname{ad}\gamma} - 1}{\operatorname{ad}\gamma} \right) \cdot e^\gamma$$

19. \checkmark The differential of $\gamma = \text{bch}(\alpha, \beta)$:

$$\delta\gamma \parallel \frac{1 - e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma} = \left(\delta\alpha \parallel \frac{1 - e^{-\operatorname{ad}\alpha}}{\operatorname{ad}\alpha} \parallel e^{-\operatorname{ad}\beta} \right) + \left(\delta\beta \parallel \frac{1 - e^{-\operatorname{ad}\beta}}{\operatorname{ad}\beta} \right)$$

20. \checkmark The differential of C :

$$\delta C_u^\gamma = \operatorname{ad}_u \left\{ \delta\gamma \parallel \frac{e^{\operatorname{ad}\gamma} - 1}{\operatorname{ad}\gamma} \parallel RC_u^{-\gamma} \right\} \parallel C_u^\gamma$$

21. \checkmark The differential of RC :

$$\delta RC_u^\gamma = RC_u^\gamma \parallel \operatorname{ad}_u \left\{ \delta\gamma \parallel \frac{1 - e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma} \parallel RC_u^\gamma \right\}$$

22. \checkmark The differential of J :

$$\delta J_u(\gamma) = \delta\gamma \parallel \frac{1 - e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma} \parallel RC_u^\gamma \parallel \operatorname{div}_u \parallel C_u^{-\gamma}$$