Schneps on "Profinite braid groups; first appearance of GT"

February-20-13
6:10 PM

For any $\mathbb{Q}$-variety/quasi-variety/scheme/stack, there is a canonical outer action

$$G_{\mathbb{Q}} \longrightarrow \mathrm{Out}(\hat{\pi}_1(X))$$

profinite completion.

univ cover $\to \tilde{X}$

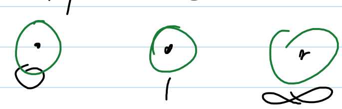
$$\begin{array}{c} \tilde{X} \\ \downarrow \quad \searrow \\ \quad\quad Y \leftarrow \text{finite cover} \\ \downarrow \quad \swarrow \\ X \end{array}$$

$$\begin{array}{c} \widetilde{\mathbb{Q}(Y)} \\ | \\ \overline{\mathbb{Q}}(Y) \\ | \; G_{\mathbb{Q}} \\ \mathbb{Q}(X) \end{array} \qquad\qquad \begin{array}{c} \widetilde{C(Y)} \\ | \\ C(Y) \\ | \\ C(X) \end{array}$$

$$\pi_1^{arith}(X) \underset{\text{often}}{=} \hat{\pi}_1(X) \rtimes G_{\mathbb{Q}}$$

4:25

Inertia groups: In a compactification of X, loops around the divisors added at ∞ generate "the inertia groups".



The action of $G_{\mathbb{Q}}$ preserves inertia up to conjugacy.

Grothendieck-Teichmuller:

1. Define braid groups
   moduli spaces / π_1
2. Define $\hat{GT}$, some properties.

3. Discuss / prove $G_{\otimes} \longrightarrow \widehat{GT}$

Artin braid groups

$$B_n = \langle \sigma_1, \dots, \sigma_{n-1} \rangle \Big/ \begin{matrix} \sigma_i\sigma_j = \sigma_j\sigma_i \quad |i-j| \geq 2 \\ \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1} \end{matrix}$$

16:03

... obvious pictures ...

$M_{0,n}$ = moduli space of genus 0 curves w/ n ordered marked points.

$M_{0,[n]} := M_{0,n} / S_n$

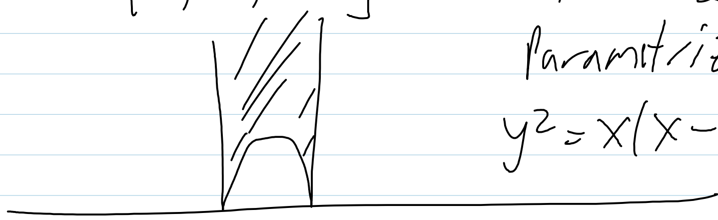
May as well take $0, 1, \infty, \tau_1 \dots \tau$

$\pi_1(M_{0,[n]}) = B_n / \langle (\sigma_1 \cdots \sigma_{n-1})^n, ? \rangle$

imprecise

$M_{0,3} = 1$ pt.

$M_{0,4} = P^1 \setminus \{0, 1, \infty\}$ $\quad \pi_1 = F_2$

$M_{0,[4]} =$ 

Parametrizes $y^2 = x(x-1)(x-\lambda)$

$\pi_1 = PSL_2(\mathbb{Z})$

$= B_3 / Z$

30:06

$K_n = \ker(B_n \longrightarrow S_n)$ "pure braids"

$\Gamma_{0,n} = \pi_1(M_{0,n})$

Claim $\Gamma_{0,n} \subset B_{n-1}/Z$ 37:30