Jones: Hochschild, loops, and Chen

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The map $HH_*(S^*(X), S^*(X)) \longrightarrow H^*(LX)$

There's a canonical map

$$\Delta^k \times LX \xrightarrow{e_k} X^{k+1}$$

Use it:

$$\rho_k : S^*(X)^{\otimes k+1} \longrightarrow S^*(X^{k+1}) \xrightarrow{e_k^*} S^*(\Delta^k \times LX)$$

$$\xrightarrow{\backslash} S^{*-k}(LX)$$

"slant product"

get

$$\delta\rho_k(\alpha_0 \otimes \dots \otimes \alpha_k) = \rho_{k-1}(b(\alpha_1 \otimes \dots \otimes \alpha_k))$$
$$\pm \rho_k(\delta(\alpha_1 \otimes \dots \otimes \alpha_k))$$

So

$$C_*(S^*(X), S^*(X)) \longrightarrow S_*^*(LX)$$

this star is the difference of the the upper stars on the left and the lower stars.

<u>Thm</u> If X is simply connected then

$$HH_*(S^*X, S^*X) \xrightarrow{\text{iso}} H^*(LX)$$

Read "Cyclic homology & equiv. homology" by Jones.

This is a version of an old result by Adams:

$$HH_*(S^*(X), \mathbb{k}) \cong H^*(\Omega X; \mathbb{k})$$

(also in the simply-connected case)

There's also

$$HH^*(S^*(X), S^*(X)) \xrightarrow[\text{iso}]{\text{not}} H_*(\Gamma(X, LX))$$

& $H^*(S^*(X), S^*(Y)) \longrightarrow H_*(\Gamma_f(Y, LX))$

(given $f: Y \to X$)