John Jones on "The Hochschild cochain complex, operads, and the Deligne conjecture II"

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6:51 AM

http://www.newton.ac.uk/programmes/GDO/seminars/2013020514001.html

Claim $\{H_*(D_n(k))\}$ is the Gerstenhaber operad G_{n-1}.

Explanation $H_*(C_2(\mathbb{R}^2)) = \begin{cases} \mathbb{Q}u & \deg 0 \\ \mathbb{Q}e & \deg n-1 \end{cases}$

u is a bilinear op of deg 0.
e — " — deg $n-1$
e is denoted $[\ ]$, w/

$$[x,y] = -(-1)^{n-1}[y,x]$$

Relations can be verified... more in video

Deligne Conjecture Mark 2.

$G_d \longrightarrow G_d^\vee$ the quadratic dual co-operad;

$G_d^\vee = \Lambda^{1-d} G_d^*$

↑ some suspension

cobar

$$F(G_d^\vee) \longrightarrow G_d$$ Kostul duality says that this is a quasi-isomorphism.

$F(G_d^\vee) = G_{d,\infty}$

Conj m2: $C^*(A,A)$ is a $G_{2,\infty}$-algebra.

(so that the induced G_{-1}-structure on $HH^*(A,A)$ is the usual one)

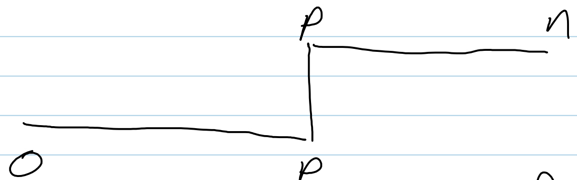
McClure-Smith on the Deligne conjecture

start from Steenrod and the theory of cup-i products.

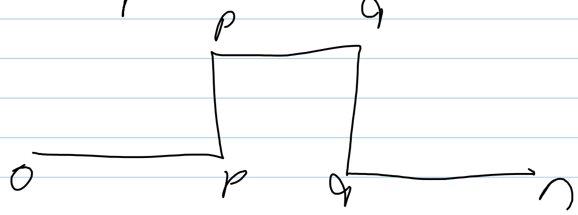
X - simplicial set   Alexander-Whitney co-product:

$$\psi_0 : C_*(X) \longrightarrow C_*(X) \otimes C_*(X)$$

$$\psi_0(\sigma) = \sum_{p=0}^{n} \sigma(0,\dots p) \otimes \sigma(p,\dots n)$$



Steenrod:



defines

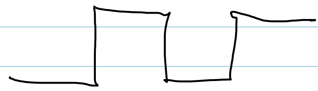
$$\psi_1(\sigma) = \sum \pm \sigma(0\dots p, q\dots n) \otimes \sigma(p\dots q)$$

$$\psi_1 : C_n(X) \longrightarrow (C_* \otimes C_*)_{n+1}$$

Steenrod gets   switch factors.

$$\partial \Psi_1(\sigma) \pm \Psi_1(\partial \sigma) = \Psi_0(\sigma) - T\,\Psi_0(\sigma)$$

So Ψ_0 is co-commutative on H.

Likewise define Ψ_2 using  ...

Now argue in the dual:

$\Psi_0 \longleftrightarrow \alpha \cup \beta$ Ψ_3 ....

$\Psi_1 \longleftrightarrow \alpha \cup_1 \beta$ Ψ_4 ...

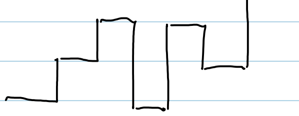
$\Psi_2 \longleftrightarrow \alpha \cup_2 \beta$

$$\partial(\alpha \cup_2 \beta) \pm (\partial\alpha) \cup \beta \pm \alpha \cup (\partial \beta) = \alpha \cup_1 \beta \pm \beta \cup_1 \alpha$$

Steenrod: Work mod 2, consider $\alpha \cup_k \alpha$,

call it $Sq^{n-k}(\alpha)$ $|k| = n$

This generalizes:

$\Gamma =$  step diagram, 4 levels, 7 steps.

$\quad\quad\quad\quad$ $\shortparallel$ $\quad$ $\shortparallel$

$\quad\quad\quad\quad$ k $\quad$ ℓ

get $\Psi_\Gamma : C_n(X) \longrightarrow \left(C_*(X)^{\otimes 4}\right)_{n+7-4}$

$\quad\quad\quad\quad$ k $\quad$ $n+\ell-k$

Step diagrams (k,ℓ) like that are

Functions $f: \underline{l} \to \underline{k}$ s.t. f is surjective, $f(i) \neq f(i+1)$ $\forall i$

These an operad make! "Step"

The tautological operad & category w/ objects $\langle n \rangle = \underline{n}$

$$\mathcal{S}(\langle n \rangle, \langle m \rangle) = \text{Functions}(\langle n \rangle, \langle m \rangle)$$

see video....

M-S prove that Step is an E_∞-operad:

1. $\text{Step}(n)$ is a free Σ_n module
2. $\text{Step}(n)$ is contractible.

---- more on video....

How close is the connection between $HH(A,A)$ and double loop spaces?

Specifically, for $HH^*(S^*(X), S^*(X))$

.... more on video.

including a mention of Chen's

iterated integrals:

There is a map

$$HH^*(S^*(X), S^*(X)) \longrightarrow H_*(\Gamma(X, LX))$$