

Non-Commutative Gaussian Elimination - The Program

Pensieve Header: Non-Commutative Gaussian Elimination - the program(s).

TheGenerators

```
n = 54;
g1 =
  Cycles[{{1, 18, 45, 28}, {2, 27, 44, 19}, {3, 36, 43, 10}, {46, 52, 54, 48}, {47, 49, 53, 51}}];
g2 = Cycles[{{7, 16, 39, 30}, {8, 25, 38, 21}, {9, 34, 37, 12},
  {13, 15, 33, 31}, {14, 24, 32, 22}}];
g3 = Cycles[{{28, 31, 34, 48}, {29, 32, 35, 47}, {30, 33, 36, 46},
  {37, 39, 45, 43}, {38, 42, 44, 40}}];
g4 = Cycles[{{1, 3, 9, 7}, {2, 6, 8, 4}, {10, 54, 16, 13}, {11, 53, 17, 14}, {12, 52, 18, 15}}];
g5 =
  Cycles[{{1, 13, 37, 46}, {4, 22, 40, 49}, {7, 31, 43, 52}, {10, 12, 30, 28}, {11, 21, 29, 19}}];
g6 = Cycles[{{3, 48, 39, 15}, {6, 51, 42, 24}, {9, 54, 45, 33}, {16, 18, 36, 34}, {17, 27, 35, 25}}];
```

TheProgram

```
 $\sigma \circ \tau := \text{PermutationProduct}[\tau, \sigma];$ 
Feed[Cycles[{}]] := Null;
Feed[ $\tau$ ] := Module[{i, j, k, l},
  i = Min[PermutationSupport[ $\tau$ ]];
  j = PermutationReplace[i,  $\tau$ ];
  If[Head[ $\sigma_{i,j}$ ] === Cycles,
    Feed[InversePermutation[ $\sigma_{i,j} \circ \tau$ ],
    (*Else*)  $\sigma_{i,j} = \tau$ ;
  For[k = 1, k < n, ++k,
    For[l = k + 1, l ≤ n, ++l,
      If[Head[ $\sigma_{k,l}$ ] === Cycles,
        Feed[ $\sigma_{i,j} \circ \sigma_{k,l}$ ]; Feed[ $\sigma_{k,l} \circ \sigma_{i,j}$ ]]
    ]
  ]
];
$RecursionLimit = ∞;
```

TheRun

```
Table[Feed[g $\alpha$ ];  $\prod_{i=1}^n (1 + \text{Count}[\text{Range}[n], j\_ /; \text{Head}[\sigma_{i,j}] == \text{Cycles}])$ , { $\alpha$ , 6}]
```

TheRun

{4, 16, 159 993 501 696 000, 21 119 142 223 872 000, 43 252 003 274 489 856 000, 43 252 003 274 489 856 000}

```
TagProperties["TheGenerators"] = {PageWidth → 9};
TagProperties["TheRun"] = {PageWidth → 12};
CellExport /@ {"TheGenerators", "TheProgram", "TheRun"}
```

```
 $\sigma \circ \tau := \text{PermutationProduct}[\tau, \sigma];$ 
Feed[Cycles[{}]] := Null;
Feed[ $\tau$ ] := Module[{i, j, k, l},
  i = Min[PermutationSupport[ $\tau$ ]];
  j = PermutationReplace[i,  $\tau$ ];
  If[Head[ $\sigma_{i,j}$ ] === Cycles,
    Feed[InversePermutation[ $\sigma_{i,j} \circ \tau$ ],
    (*Else*)  $\sigma_{i,j} = \tau$ ;
  For[k = 1, k < n, ++k,
    For[l = k + 1, l ≤ n, ++l,
      If[Head[ $\sigma_{k,l}$ ] === Cycles,
        Feed[ $\sigma_{i,j} \circ \sigma_{k,l}$ ]; Feed[ $\sigma_{k,l} \circ \sigma_{i,j}$ ]]
    ]
  ]
];
$RecursionLimit = ∞;
```

43 252 003 274 489 856 000 / (8! * 3^8 * 12! * 2^12)

```
Export["TGraph.png", ImageCrop[
  Rasterize[Graphics[{
    EdgeForm[Thin],
    Table[
      If[Head[σi,j] === Cycles || i == j,
        {Red, Rectangle[{i - 1, n - j}, {i, n - j + 1}]},
        {Yellow, Rectangle[{i - 1, n - j}, {i, n - j + 1}]}}
      ], {i, n}, {j, i, n}
    ]
  }, AspectRatio → 1 / GoldenRatio, ImageSize → 720], RasterSize → 720]
]]
TGraph.png
```

```

SetDirectory["C:/drorbn/AcademicPensieve/2013-01/NCGE"];

TagProperties[_] := {};
Options[CellExport] = {
  PageWidth → 6, CellFilter → Identity,
  ExportFormat → ".png", ExportOptions → {}, Split → False
};
CellExport[tag_String] := CellExport[tag, TagProperties[tag]];
CellExport[tag_String, OptionsPattern[]] := Module[
  {cells, cell},
  cells = OptionValue[CellFilter][Cases[
    NotebookGet[EvaluationNotebook[]],
    c_Cell /; FreeQ[List@@c, Cell] && !FreeQ[c, CellTags → tag],
    Infinity
  ]];
  If[!OptionValue[Split],
    If[Length[cells] ≥ 1,
      If[Length[cells] == 1,
        cells = Append[First[cells], PageWidth → 72 OptionValue[PageWidth]],
        cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
      ];
    Show[
      Import[Export[tag <> OptionValue[ExportFormat],
        cells, Sequence @@ OptionValue[ExportOptions]]],
      ImageSize → 120
    ]
  ],
  k = 0;
  GraphicsRow[Table[
    ++k;
    Show[
      Import[Export[
        tag <> "-" <> ToString[k] <> OptionValue[ExportFormat],
        Append[cell, PageWidth → 72 OptionValue[PageWidth]],
        Sequence @@ OptionValue[ExportOptions]
      ]],
      ImageSize → 120
    ],
    {cell, cells}
  ]]
]
];

```