

$$\Theta = P dx + Q dy$$

$$P, Q \in \mathbb{R}[x, y]$$

$$\deg P, Q \leq n$$

$P, Q$  must be co-prime.

or

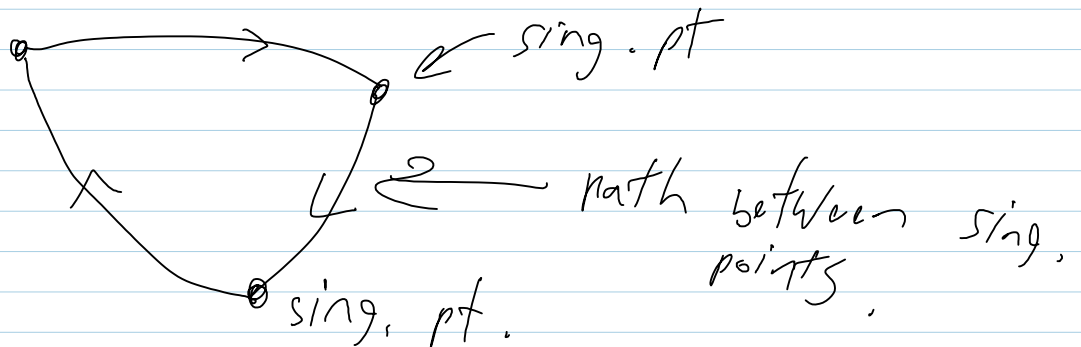
$$\dot{x} = Q(x, y)$$

$$\dot{y} = -P(x, y)$$

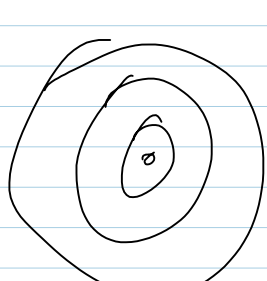
The  $w$ -limit of a trajectory is the set of all limits  $\lim x(t_n)$  for some  $t_n \rightarrow \infty$ .

Poincaré-Bendixon In the plane an

$w$ -limit set is either a polycycle:

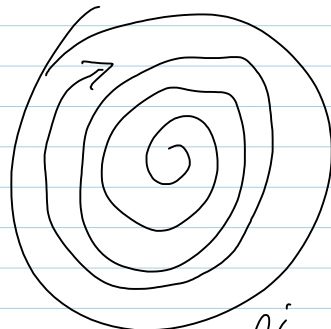


or a periodic trajectory; this has two cases:



integrable

and



limit cycle.

Hilbert 6th problem: How many limit cycles may appear in a system of degree  $n$ ?

Thm (very hard, Ilyashenko, Ecalle) Every specific system has finitely many limit cycles

Even for quadratic systems no uniform bound is known! Currently the largest number known is 4.