Knot Question by Alon Amit

What are these knots?

Wed, Jun 27, 2012 at 9:31 PM

To: drorbn@math.toronto.edu

Hi Dror!

http://en.wikipedia.org/wiki/Discipline_(King_Crimson_album)

There's two knots here (well, links, but let's ignore the outer unknot). What are they? Is there a good way to address this question with a computer and without a piece of string and lots of manual labor?

Best regards :-)

Alon

Alon,

As is, the answer to "what are they?" is "they are what they are". Knot enumeration goes up to about 17 crossings; these knots have way more, so they are not in the tables, hence they have no name or serial number.

Yet you can ask several other questions, and maybe also answer:

1. Can these knots be written in using fewer crossings (in particular, are they trivial)? --- The first knot is "alternating" (the crossings come over-under-over-under as you travel along the knot). By a standard theorem, alternating knots that have no "nugatory" crossings cannot be simplified. Never mind what nugatory means, the first knot clearly doesn't have any nugatory crossings, so it follows that it cannot be simplified. The second knot is not alternating (find where!), yet by a very simple simplification it can be made alternating. Hence after that simple simplification it cannot be simplified any further.
2. What are the Jones and Alexander polynomials of these knots? --- There are standard software packages to compute these and similar invariants (e.g. http://katlas.org/wiki/The_Mathematica_Package_KnotTheory%60) so the answer is easy to obtain, but why bother? You'll get some lengthy polynomials.
3. What are the symmetry groups of these knots? They have some obvious symmetries, but perhaps they also have some non-obvious ones? --- You'd probably want to use the program SnapPea, http://en.wikipedia.org/wiki/SnapPea, but you may find it a challenge.

Best,

Dror.