Valeriy Bardakov: Residual properties for virtual and welded braids

June-08-12
3:54 AM

I. The braid group B_n

1. Normal form (Artin-Markov)

$$\pi : B_n \longrightarrow S_n$$

$$\ker \pi = PB_n$$

$\varphi_n : PB_n \longrightarrow PB_{n-1}$ "delete strand n".

$\ker \varphi_n = U_n$ is a free group and

$$PB_n = U_n \rtimes P_{n-1}$$
$$= U_n \rtimes (U_{n-1} \rtimes \ldots \quad)$$

So if $\beta \in PB_n$,

$$\beta = U_2 U_3 \cdots U_n \cdot \beta_\pi$$

$$\pi^{-1}(S_n)$$

2. Rep. by automorphisms of $F_n = \langle x_1 \ldots x_n \rangle$

$$\rho_A : B_n \longrightarrow \mathrm{Aut}(F_n), \quad \ker \rho_A = 1,$$

$\psi \in \mathrm{Aut}\, F_n$ is in the image of ρ_A iff

1. $\psi(x_i) = f_i x_{\tau(i)} f_i^{-1}$
2. $\psi(x_1 \ldots x_n) = x_1 \ldots x_n$

3. Linear representations

$\rho_{LKB} : B_n \longrightarrow GL_m(\mathbb{Z}[t^{\pm 1}, q^{\pm 1}])$ is faithful.

Lawrence Krammer Bigelow

II. Some generalizations of B_n:

1. Group of conjugating automorphisms:
   - drop the second cond. from the image of ρ_A; call it C_n.

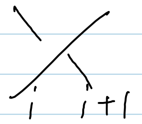
⋮

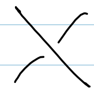
<u>Def</u> G is residually torsion-free nilpotent if $\gamma_1 = G \supset \gamma_2 = [\gamma_1, G] \supset \gamma_3 = [\gamma_2, G] \supset \cdots$

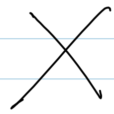
1. G/γ_i is torsion free nilpotent group.
2. $\bigcap_{i=1}^{\infty} \gamma_i = 1.$

---

vB_n & wB_n:

σ_i : 

σ_i^{-1}: 

ρ_i :  $\rho_i^2 = 1$

$\sigma_i \rho_{i+1} \rho_i = \rho_i \rho_{i+1} \sigma_i$ defines vB_n.

$wB_n = vB_n / \sigma_i \sigma_{i+1} \rho_i = \rho_{i+1} \sigma_i \sigma_{i+1}$

⋮

there are also vP_n & wP_n.

Problems for $G_n \in$ {the above groups}

1. Normal forms for words?
2. $G_n \leq Aut(F_n)$?
3. is G_n linear?

III wB_n: FRR proved that $wB_n \cong C_n$

$$\epsilon_{ij} : \begin{cases} x_i \longmapsto x_j^{-1} x_i x_j \\ x_k \longmapsto x_k \end{cases}$$

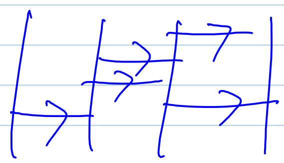
Thm $wP_n \cong D_n \ltimes wP_{n-1} \cong \dots$
$D_n \ltimes (D_{n-1} \ltimes (\dots))$

where

$$D_i = \langle \underbrace{\epsilon_{1i} \dots \epsilon_{i-1,i}}_{\mathbb{Z}^{i-1}} , \underbrace{\epsilon_{i,1} \dots \epsilon_{i,i-1}}_{F_{i-1}} \rangle \Big/ \text{infinitely many relations.}$$

$wP_n^+ := D_n^+ \ltimes (D_{n-1}^+ \dots)$

where D_n^+ is $\langle \epsilon_{i1}, \dots \epsilon_{i,i-1} \rangle \cong F_{i-1}$

(i.e.  all arrows go right)

Question Is $wP_n^+ \cong wP_n$?
(CPVW)

Thm (Bardakov, Mikhailov) No.
(by $\Delta(wP_4^+) \neq \Delta(P_4)$)

---

3. Linear reps:

Burau: $\rho_B : B_n \to GL_n(\mathbb{Z}[t^{\pm 1}])$

Gassner: $\rho_G : P_n \to GL_n(\mathbb{Z}[t_1^{\pm 1} \dots t_n^{\pm}])$

Both extend to wP_n ....

$\boxed{\text{Thm } \rho_G \text{ is not faithful on } wP_2.}$?

$\boxed{\textbf{Thm } \rho_G \text{ is not faithful on } wP_2.}$?

Prop There is a rep. $\rho_{LKB} : wB_n \to GL_m(\mathbb{Z}[t^{\pm 1}, q^{\pm 1}])$

That extends the classical LKB rep.

Question is it faithful?

Problem

automorphisms of F_n w/ trivial Abelianization.

$$\mathrm{Aut}(F_n) \geq IA(F_n) \geq wP_n \geq P_n \quad n \geq 3$$

non-linear   non-lin   ?   linear

IV vB_n

1. Normal Form

Thm $VP_n = V_n \rtimes (V_{n-1} \rtimes \dots \quad)$

is V_n free?

Prop (BMVW) 1. $VP_3 = H_3 * \mathbb{Z}$

2. $H_3 \leq wP_3$

3. $H_3 = Q_3 \rtimes \mathbb{Z}$ Q_3 has ∞-many relations.

$\exists : 1 \to F_\infty \to Q_3 \to \mathbb{Z}^4 \to 1$

Thm $VP_3 = (F_3 \rtimes F_2) * \mathbb{Z}$

Problem 1. Find normal form for vP_n, $n > 3$

2. Linearity

3. As Bellingeri said,

$$\exists \rho : vB_n \longrightarrow \mathrm{Aut}(F_{n+1}) \quad \mathrm{Ker} = ?$$