

$$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$$

$$\text{Simplify}\left[b_2 \text{Det}\left[\begin{pmatrix} d_{11} & c_1 \\ d_{21} & c_2 \end{pmatrix}\right] + b_1 \text{Det}\left[\begin{pmatrix} c_1 & d_{12} \\ c_2 & d_{22} \end{pmatrix}\right]\right]$$

$$b_2 c_2 d_{11} - b_1 c_2 d_{12} - b_2 c_1 d_{21} + b_1 c_1 d_{22}$$

Factor[%]

$$b_2 c_2 d_{11} - b_1 c_2 d_{12} - b_2 c_1 d_{21} + b_1 c_1 d_{22}$$