Hambleton: Coarse geometry and group actions

jW/ Lucian Sawin.

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4:10 PM

$G \times X \to X$ a group action

$X^G = \{x \in X : \forall g \in G \ gx = x\} \overset{?}{\longleftrightarrow}$ topology of X

P.A. Smith Theory: If $G = \mathbb{Z}/p$ acts on a f.d. space X which is a $\mathbb{Z}/p$-homology-sphere, then X^G is a $\mathbb{Z}/p$ homology sphere.

Same w/ sphere $\to$ disk

Question If $X = S^n$, does X^G have to be a sphere too? How is X^G embedded in X?

Example The Alexander Horned Sphere can be the fixed point set of a $\mathbb{Z}/2$ action on S^3.

Kister's Thm $\exists$ a smooth $\mathbb{Z}/6$-action (or $\mathbb{Z}/pq$, p,q distinct primes) on $\mathbb{R}^n$, $n \geq 8$, with no fixed points.

Coarse Geometry Gromov: "large scale geometry"

(X,d) metric space "proper": closed balls are compact.

$f : X \xrightarrow{\text{not nec. continuous.}} Y$ is is a "Coarse map" if

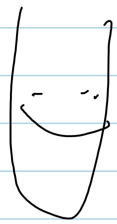
1. $f^{-1}(\text{bndd set}) = \text{bndd}$.

2. $\forall R>0\ \exists S$ s.t. $d(x_1,x_2)<R$
$\Longrightarrow d(fx_1,fx_2)<S$.

"Close" coarse maps; "coarse equivalence."

Example. $f: X \to Y$ is an (L,C)-quasi-isometry if

1. $\frac{1}{C}d(x_1,x_2)-L \leq d(fx_1,fx_2) < C\,d(x_1,x_2)+L$
2. $N_L(f(X))=Y$

$\mathbb{Z}^n \subset \mathbb{R}^n$ is a quasi-isometry, but not  $\subset \mathbb{R}^3$ to $\mathbb{R}^2$

"Coarse group action", "co-compact", "properly discontinuous":

$\forall R>0\ \exists M>0\ \forall x_1,x_2$

$$\#\{g\in G \mid gN_R(x_1)\cap N_R(x_2)\neq\emptyset\} \leq M$$

Milnor–Svarc: Let (X,d) be a proper-geodetic metric space. If Γ has a proper cobounded coarse action on X, then Γ

is f.g. and $X \sim (\Gamma$, word metric$)$

Kleiner–Leeb: a coarse action is coarsly equiv. to an isometric action. 4:41

$$\xrightarrow[\text{equiv.}]{\text{coarsly}}$$

Yet the left space has a f.p. free involution [but this action is coarsly ineffective]

⋮ Lost wand 4:45.