Coleman Lecture 7: Time Reversal, Dyson's Formula

November-24-11
12:23 PM

Time Reversal. $H = \frac{p^2}{2m} + V(x)$

$$q(t) \mapsto q(-t)$$
$$p(t) \longmapsto -p(-t)$$

Cannot be implemented by a unitary transformation!

$$\left.\begin{array}{lcl} q(-t) & & U_t^{*} q(t) U_t \\ -p(-t) & \neq & U_t^{*} p(t) U_t \end{array}\right\}$$ This would contradict the CCR

An anti-unitary operator:

$$U(\alpha a + \beta b) = \alpha U(a) + \beta U(b)$$

becomes:

$$\Omega(\alpha a + \beta b) = \bar{\alpha}\, \Omega(a) + \bar{\beta}\, \Omega(b)$$

$$(\Omega a, \Omega b) = \overline{(a, b)} = (b, a)$$

Can find Ω s.t.

$$q(-t) = \Omega^{-1} q(t) \Omega$$
$$-p(-t) = \Omega^{-1} p(t) \Omega$$

In fact, on wave functions, Ω is just complex conjugation.

Now $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \mu\phi^2$

$$\Omega_{p.t} |p_1 \ldots p_N\rangle = |p_1 \ldots p_N\rangle$$

Parity time reversal

$$\Omega_{PT}\, a_{\vec{p}} = a_{\vec{p}}\, \Omega_{PT} \qquad \Omega_{PT}\, a^*_{\vec{p}}\, \Omega^{-1}_{PT} = a^*_{\vec{p}}$$

$$\phi(x) = \int \frac{1}{\sqrt{\;}} \underline{\qquad\qquad}$$

$$\Omega^{-1}_{PT}\, \phi(x)\, \Omega_{PT} = \phi(-x)$$

Perturbation Theory. For the rest of that course..... ~21:00

Schrödinger Picture

The observables p, q are constant.

$$i\frac{d}{dt}\Psi = H(\Psi)$$

$$\Psi(t) = U_{t,t'}\Psi(t')$$

$U_{t,t'}$ is linear & unitary: $U^*_{t't} = U_{tt'}$,

There's a composition property.

$$i\frac{d}{dt}U_{t,t'} = HU_{t,t'} \qquad U_{t,t} = I$$

Heisenberg Picture

States are constant,

$$q(t) = U^{*}_{t,0}\, q(0)\, U_{t,0} \quad \text{etc.}$$

If H is time independent,

$$U(t,t') = e^{-iH(t-t')}$$

Assume now,

$$H(q,p,t) = H_0(p,q) + H'(p,q,t)$$

The interaction picture:

$$\begin{pmatrix} q_I(t) \\ p_I(t) \end{pmatrix} = e^{iH_0 t} \begin{pmatrix} q(0) \\ p(0) \end{pmatrix} e^{-iH_0 t}$$

$$\Psi_I(t) = e^{iH_0 t}\, \Psi_S(t)$$

$$i\frac{d}{dt}\Psi_I(t) = e^{iH_0 t}\left(-H_0 \Psi_S + H \Psi_S\right)$$

$$= \ldots = H_I(t)\, \Psi_I(t)$$

<u>Dyson's formula</u> (for $t > t'$)

$$U_I(t,t') = T\,\mathrm{Exp}\left(-i\int_{t'}^{t} dt''\, H_I(t'')\right)$$

57:30