Coleman Lecture 6: Symmetries and Conservation Laws, II

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Internal Symmetries.

Example.

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^1 \partial^\mu \phi^1 + \frac{1}{2}\partial_\mu \phi^2 \partial^\mu \phi^2$$

has SO(2) symmetry

$$\lambda: \quad \begin{aligned} \phi^1 &\mapsto \phi^1 \cos\lambda + \phi^2 \sin\lambda \\ \phi^2 &\mapsto -\phi^1 \sin\lambda + \phi^2 \cos\lambda \end{aligned} \qquad \mathcal{L} \mapsto \mathcal{L}$$

could add an SO(2)-invariant interaction.

$$D\phi^1 = \phi^2 \qquad D\phi^2 = -\phi^1$$

$$\pi_1^\mu = \partial^\mu \phi^1 \quad \pi_2^\mu = \partial^\mu \phi^2$$

$$F^\mu = 0$$

$$J^\mu = \pi_a^\mu D\phi^a - F$$

$$= (\partial^\mu \phi^1)\phi^2 - (\partial^\mu \phi^2)\phi^1$$

$$Q = \int d^3x\, J^0 = \int d^3x\, \left((\partial^0 \phi^1)\phi^2 - (\partial^0 \phi^2)\phi^1\right)$$

$$\phi(x,t) = \int \frac{d^3k}{\cdots} e^{ik\cdot x}(\ \cdots\ )$$

$$Q = \int \frac{d^3k}{2\omega_k}\left[\ \cdots\ \right]$$

Define $b_k = \frac{a_k^{(1)} + i a_k^{(2)*}}{\sqrt{2}}$ $c_k = \frac{a_k^{(1)} - i a_k^{(2)*}}{\sqrt{2}}$

$$[b_k, b_{k'}^*] = [c_k, c_{k'}^*] = \delta^3(k-k')$$

$$Q = \int d^3\vec{k} \left[b_k^* b_k - c_k^* c_k \right]$$

So the "Q-charge" counts "b-particles" minus "c-particles".

$$H = \int d^3k \; \vec{k}\, \omega_{\vec{k}} \left[b_k^* b_k + c_k^* c_k \right]$$ 23:48

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Complex Fields

$$\psi = (\phi^{(1)} + i\phi^{(2)})/\sqrt{2}$$
$$\psi^* = (\phi^{(1)} - i\phi^{(2)})/\sqrt{2}$$

$$\psi = \int d^3k \frac{1}{\sqrt{\quad}} \left(e^{i \cdots} b_k + \cdots c^* \right)$$

ψ & ψ^* obey the Klein-Gordon eq'n.

$[\psi(\vec{x}), \psi(\vec{y})] = 0$ at equal times

$$[\psi(x,t), \partial_0 \psi(y,t)] = 0$$
$$[\psi(x,t), \partial_0 \psi^*(y,t)] = i\delta^3(x-y)$$

$$\mathcal{L} = \partial^\mu \psi \, \partial_\mu \psi^* - m^2 \psi^* \psi$$

In general:

$$\mathcal{L} = \mathcal{L}(\psi, \psi^*, \partial_\mu \psi, \partial_\mu \psi^*) \quad \text{s.t.}$$

$$S = \int d^4x \, \mathcal{L} = S^*$$

$$\delta S = \int d^4x \left(\underline{X} \, \delta\psi + \underline{X}^* \delta\psi^* \right)$$

treating $\delta\psi$ & $\delta\psi^*$ as independent, get

$$\underline{X} = 0, \quad \underline{X}^* = 0$$

but this is false.

Yet, choose $\delta\psi = \delta\psi^*$ & get

$$\underline{X} + \underline{X}^* = 0$$

Choose $\delta\psi = -\delta^*\psi$ & get

$$\underline{X} - \underline{X}^* = 0$$

So we got the same thing.

λ:

$$\psi \longmapsto e^{i\lambda} \psi \qquad \psi^* \longmapsto e^{-i\lambda} \psi^*$$

So

$$D\psi = i\psi \qquad D\psi^* = -i\psi^*$$

$$\Pi_\psi{}^\mu = \partial^\mu \psi^* \qquad \Pi_{\psi^*}{}^\mu = \partial^\mu \psi$$

$$F^{\mu} = 0$$

So $\quad J^{\mu} = \ldots =$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi^a \partial^\mu \phi^a - \frac{\mu^2}{2}\phi^a\phi^a\right) \qquad a = 1, \ldots, n$$

is invariant under $SO(n)$ ... $\frac{n(n-1)}{2}$ cons. currents:

$$j^{\mu(ab)} = (\partial^\mu \phi^a)\phi^b - (\partial^\mu \phi^b)\phi^a$$

A discussion of the invariance of charge, as in BBS/today.

Discrete Symmetries.

$$\phi^a(x) \longmapsto \phi'^a(x) \quad \text{s.t.}$$

$$\int \mathcal{L} \quad \text{is invariant.}$$

Example $\quad \mathcal{L} = \frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a - \frac{\mu^2}{2}\phi^a\phi^a$

$a = 1, 2$

under $\phi^1 \longrightarrow \phi^1, \quad \phi^2 \longrightarrow -\phi^2$

is there a unitary trans. s.t.

$$\phi^1 \to U^{-1}\phi^1 U \qquad \phi^2 \longmapsto -U^{*}\phi^2 U$$

$\longrightarrow$ U multiplies by $(-1)^{\#\text{ of } \phi 2 \text{ particles}}$

In the b-c language, $U^* b_k U = c_k$, $U^* c_k U = b_k$. This is charge conjugation!

In this case, $U = U^*$ so U is both unitary and Hermitian.

Then a discussion of parities.....