

The construction of a quantum field within Fock space.

$$\phi(x) = \phi_- + \phi_+ \quad \alpha = \sqrt{(2\pi)^3 2\omega_p} \cdot a$$

$$\phi_-(x) = \int \frac{d^3p}{(2\pi)^3 2\omega_p} a_p e^{-ipx}$$

$$\phi_-^* = \phi_+$$

$$\phi_+(x) = \int \frac{d^3p}{(2\pi)^3 2\omega_p} a_p^* e^{+ipx}$$

$$[\phi_+(x), \phi_-(y)] = \Delta_+(x-y) = \int \frac{d^3p}{(2\pi)^3 2\omega_p} e^{-ip(x-y)}$$

$\Delta_+(x) \stackrel{?}{=} 0$ when $x^2 < 0$? No!

Take $\phi = \phi_+ + \phi_-$; is ϕ a quantum field?

$$[\phi(x), \phi(y)] = \Delta_+(x-y) - \Delta_+(y-x) =: i\Delta(x-y)$$

(1) $(\square^2 + m^2)\phi(x) = 0$

The "Klein-Gordon Eqn"
First written by
Schrödinger.

(2) $[\phi(x), \phi(y)] = i\Delta(x-y, m^2)$

(3) ϕ is hermitian.

From these equations we can recover

a_p & a_p^\dagger & all of Fock space.

(2) can be replaced with

$$(2'a) [\phi(t, \vec{x}), \phi(t, \vec{y})] = 0$$

$$(2'b) [\phi(t, \vec{x}), \dot{\phi}(t, \vec{y})] = i \int^3 (\vec{x} - \vec{y})$$