Lecture 2 Clippings

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Clippings from http://www.math.toronto.edu/drorbn/LOP.html#pcs

$$\mathcal{Z} = \int_{\mathbf{R}^n} d\vec{x} e^{it(\frac{1}{2}\lambda_{ij}x^ix^j + \lambda_{ijk}x^ix^jx^k)} \delta^l(F(\vec{x})) \det\left(\frac{\partial F^a}{\partial \mathcal{G}_b}\right)(\vec{x}).$$

$$\delta^l(F(\vec{x})) = \frac{1}{(2\pi)^l}\int_{\mathbf{R}^l} d^l\phi e^{iF^a(\vec{x})\phi_a}$$

$$\det\left(J_0 + \frac{1}{\sqrt{t}}J_1(\vec{x})\right) = \det(J_0)\sum_m \left(\frac{1}{\sqrt{t}}\right)^m \mathrm{Tr}(\Lambda^m J_0^{-1})(\Lambda^m J_1(\vec{x})).$$

$$\int d^l\bar{c}d^lc e^{i\bar{c}_a J^a{}_b c^b} \propto \det(J).$$

$$\mathcal{Z} \propto \int_{\mathbf{R}^n} d\vec{x}\frac{1}{(2\pi)^l}\int_{\mathbf{R}^l} d^l\phi \int d^l\bar{c}d^lc e^{i(t(\frac{1}{2}\lambda_{ij}x^ix^j+\lambda_{ijk}x^ix^jx^k)+F^a(\vec{x})\phi_a+\bar{c}_a(\frac{\partial F^a}{\partial \mathcal{G}_b})c^b)}$$

$$\delta A = -D^A c = -(dc + [A,c])$$

$$\frac{k}{4\pi}D_i^A B^i = 0$$

$$\begin{aligned}\mathcal{L}_{tot}(B,\phi,c,\bar{c}) &= \mathcal{L} + \mathcal{L}_{gauge\text{-}fixing} + \mathcal{L}_{ghosts} = \\ &= \frac{k}{4\pi}\int_{M^3} tr(B\wedge D^A B + \frac{2}{3}B\wedge B\wedge B) + \\ &\quad + \frac{k}{4\pi}\int_{M^3} tr(\phi D_i^A B^i) + \\ &\quad + \frac{k}{4\pi}\int_{M^3} tr\bar{c}D_i^A(D_A^i + \mathrm{ad}\, B^i)c\end{aligned}$$

Clippings from my Thesis,

http://www.math.toronto.edu/drorbn/LOP.html#thesis

$$\mathcal{O}_{X,R}(A) = tr_R\,\mathcal{P}\exp\left(\int ds\dot{X}^i(s)A_i(X(s))\right) = \dim R - \int ds\dot{X}^i(s)A_i^a(X(s))R^\alpha_{a\alpha} + \int_{s_1<s_2} ds_{1,2}\dot{X}^{i_1}(s_1)\dot{X}^{i_2}(s_2)A_{i_1}^{a_1}(X(s_1))A_{i_2}^{a_2}(X(s_2))R^{\alpha_1}_{a_1\alpha_2}R^{\alpha_2}_{a_2\alpha_1} - \cdots . \quad (1.3)$$

$$\int_{\mathbf{R}^N} d^N x\, e^{ik\mathcal{L}} \underset{k\to\infty}{\sim} \sum_{i=1}^{I} \frac{e^{\frac{i\pi}{4}\operatorname{sign} L(x_i)}}{\sqrt{(4\pi k)^N\,|\det L(x_i)|}} e^{ik\mathcal{L}(x_i)}$$

(a) For each gauge propagator in D, marked, say, as $\overset{i,a}{\underset{x}{}}$ — — $\overset{i',a'}{\underset{y}{}}$ take the term

$$V^{aa'}_{ii'}(x,y). \quad (2.1)$$

$V^{ab}_{IJ}(x,y)$ is defined to be the inverse of the bosonic free part of the Lagrangian $\mathcal{L}$. The symbols "I" and "J" are either numbers i,j in the range $1-3$, or the symbol ϕ, and with this understood V is defined by the relations: (the differentiations[4] are all with respect to x.)

$$\begin{aligned}
t_{ab}D_i\sqrt{g}g^{ij}V^{bc}_{j\phi}(x,y) &= 2\pi i\delta^c_a\delta(x,y),\\
t_{ab}D_i\sqrt{g}g^{ij}V^{bc}_{jk}(x,y) &= 0,\\
t_{ab}\left(\epsilon^{ijk}D_jV^{bc}_{kl}(x,y) + D^iV^{bc}_{\phi l}(x,y)\right) &= 2\pi i\delta^c_a\delta^i_l\delta(x,y),\\
t_{ab}\left(\epsilon^{ijk}D_jV^{bc}_{k\phi}(x,y) + D^iV^{bc}_{\phi\phi}(x,y)\right) &= 0.
\end{aligned}$$

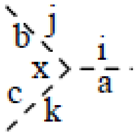
(b) For each ghost propagator in D, marked, say, as $\underset{y}{d'}$ ———— $\underset{z}{d}$ take the term

$$G^{d'd}(y,z).$$

G is defined to be the inverse of the ghost free part of the Lagrangian $\mathcal{L}$ — that is to say, it is defined by the relation: (the differentiations are all with respect to x.)

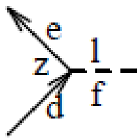
$$t_{ab}D_iD^iG^{bc} = -2\pi\delta_a^c\delta(x,y).$$

(c) For each marked A^3 vertex in D use the rule



$$\longrightarrow \frac{i}{2\pi}\int_{M^3} dx\, t_{abc}\epsilon^{ijk}. \tag{2.2}$$

(d) For each $\bar{c}Ac$ vertex in D use the rule

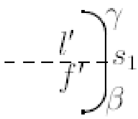


$$\longrightarrow \frac{1}{2\pi}\int_{M^3} dz\, t_{dfe}D_z^l. \tag{2.3}$$

Here D_z^l denotes differentiation with respect to z^l,

$$D_z^l = \sqrt{g}g^{lm}\frac{D}{Dz^m}$$

acting only on the z-dependence of the term coming from the ghost propagator leaving the vertex. For a better understanding, let us look at this term



$$\longrightarrow -R^{\beta}_{f'\gamma}\dot{X}^{l'}(s_1).$$

$$G^{ab}_{\Delta}(x,y) = \frac{t^{ab}}{4\pi|x-y|}$$

$$\underset{a,i}{\overset{x}{-}} - - \underset{b,j}{\overset{y}{-}} V^{ab}_{ij}(x,y) = \langle A_i^a(x)A_j^b(y)\rangle = 2\pi i\epsilon_{ikj}\partial_x^k\frac{t^{ab}}{4\pi|x-y|} = \epsilon_{ijk}t^{ab}\frac{i(x-y)^k}{2|x-y|^3}.$$