Braverman Lecture 2

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12:56 PM

Vector bundles, sections , pullback bundles, hom of two bundles, elliptic operators by their symbols

Theorem:

1. If $\mathcal{D}$ is elliptic then it is Fredholm.
2. $ind_G(\mathcal{D})$ depends only on the homotopy class of the leading symbol of $\mathcal{D}$.

$\sigma_L(\mathcal{D}) \longrightarrow$ element $K_G(T^*M) \xrightarrow{t\text{-ind}_G} R(G)$

the leading symbol of $\mathcal{D}$ — topological index

Theorem. $t\text{-ind}_G(\sigma_L(\mathcal{D})) = ind_G(\mathcal{D})$

Def. of $K(X)$: formal differences of vector bundles. Likewise $K_G(X)$

$K(\cdot) = \mathbb{Z}$ $\quad K_G(\cdot) = R(G)$

$K(S^1) = \mathbb{Z}$

$K(S^2) = \mathbb{Z} \oplus \mathbb{Z}$

If X is non-compact, let $\bar{X}$ be it's 1-pt compactification & set (w/ 1-pt $= \infty$)

$$K(X) := K(\bar{X}) / K(\infty)$$

Morphisms in the non-compact case:

$E \xrightarrow{\sigma} F$ s.t. σ is invertible outside of a compact set.

Thus $\sigma_L(\mathcal{D})$ is an element of $K_G(T^*M)$.

$t\text{-ind}: K(T^*M) \to \mathbb{Z}$ defined by:

$M \subset \mathbb{R}^n$ by Whitney so $T^*M \subset \mathbb{R}^n \oplus \mathbb{R}^n = \mathbb{C}^n$.

Given $f: M \to N$ have

$M \subset \mathbb{R}^n$ by Whitney so

$T^*M \subset \mathbb{R}^n \oplus \mathbb{R}^n = \mathbb{C}^n$.

⋮

$f: M \to N$ have $f^*: K(N) \to K(M)$

Can construct $K(T^*M) \xrightarrow{j_*} K(\mathbb{C}^n) \cong K(\bullet) = \mathbb{Z}$

Bott periodicity

Everything generalizes to the equivariant situation.

The proof of A-S works by listing properties of both sides and showing that there is a unique object having these properties. Without a G there are not enough properties to do that.