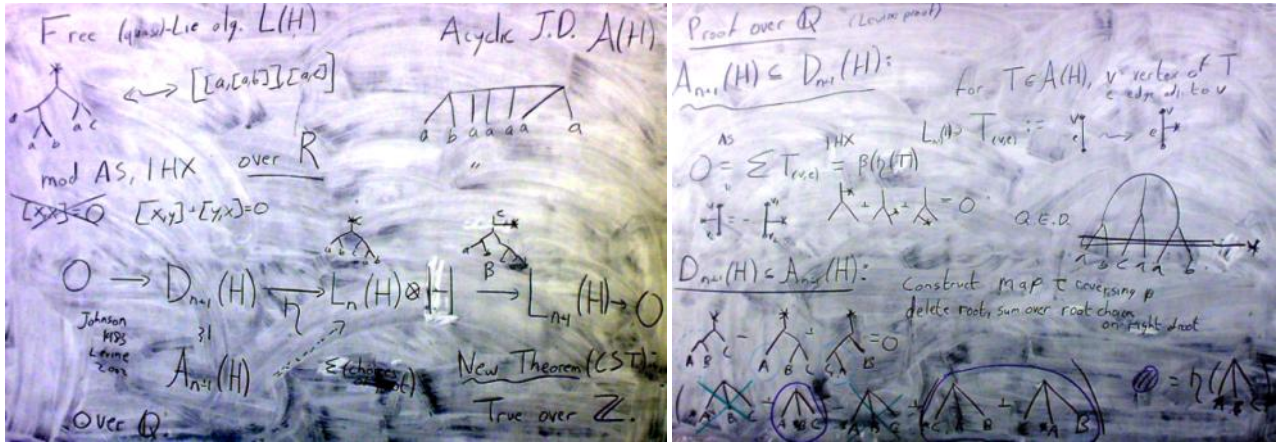


# The famed "Drinfel'd Lemma", following Moskovich

February-23-11  
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Thm.  $\mathcal{S}der_n \cong T_n \leftarrow \begin{matrix} \text{unrooted trees} \\ \text{on } n \text{ letters} / \text{AS, IHX.} \end{matrix}$

$$0 \rightarrow T_n \xrightleftharpoons[\beta]{\alpha} \mathcal{S}der_n \xrightarrow{\gamma} Lie_n \rightarrow 0$$

$$\tau: Lie_n \rightarrow \frac{\mathcal{S}der_n}{\alpha(T_n)} \xrightarrow{\gamma \circ \beta} \frac{\mathcal{S}der_n}{0}$$

Given  $\alpha, \beta, \tau, \gamma$  how make a homotopy?

Eg.  $0 \rightarrow \mathbb{Z} \xrightarrow{\alpha} \mathbb{Z} \rightarrow \mathbb{Z}/2 \rightarrow 0$

$$D[a,b] := [Da, b] - [a, Db] \quad D[b,a] = [Db, a] - [b, Da]$$

$$D[[a,b], c] = [[Da, b], c] - [[a, Db], c] - [[a,b], Dc]$$

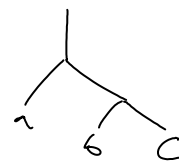
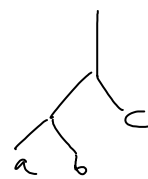
$$\tau[a,b] = [Da, b] \quad D[a,b] = [Da, b] + [a, Db]$$

$$Dx =$$

$$\tau[[a,b], c] = a \otimes [c, b] + b \otimes [a, c]$$

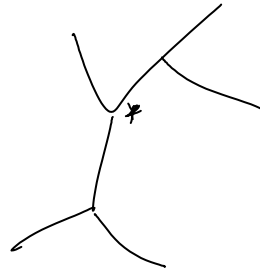
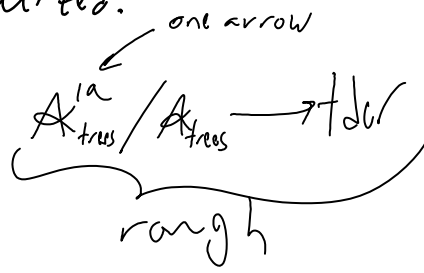
$$\tau[a, [b, c]] = a \otimes [c, b]$$

$$a \otimes [b, c] + b \otimes [c, a] + c \otimes [a, b]$$



Now localized.

claim



$$I_* = H_* - \text{crossed tree}_*$$

↓ ↓ ?  
0

$$\begin{array}{c}
 + \\
 \diagdown \quad \diagup \\
 \quad \quad \quad + \\
 \diagup \quad \diagdown \\
 +
 \end{array}
 +
 \begin{array}{c}
 + \\
 \diagdown \quad \diagup \\
 \quad \quad \quad - \\
 \diagup \quad \diagdown \\
 +
 \end{array}
 +
 \begin{array}{c}
 - \\
 \diagdown \quad \diagup \\
 \quad \quad \quad + \\
 \diagup \quad \diagdown \\
 +
 \end{array}
 = 0$$

⇒ To get  $IHX$  on the nose,  $\tau$  would have to "average" ends, not just add them.