Tail scattering towards the head

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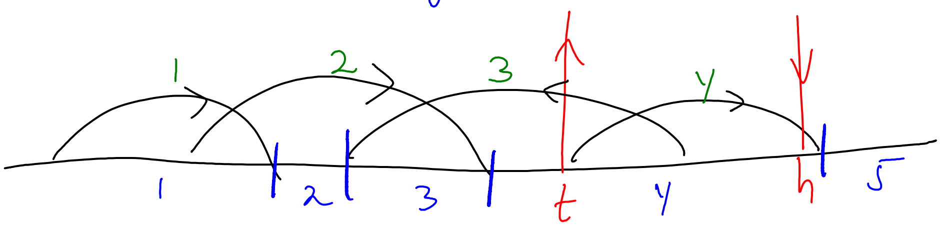
12:57 PM

cut by arrow heads:



There must be a good way of formalizing "moving targets".

$$R\Lambda + M\phi = 0,$$

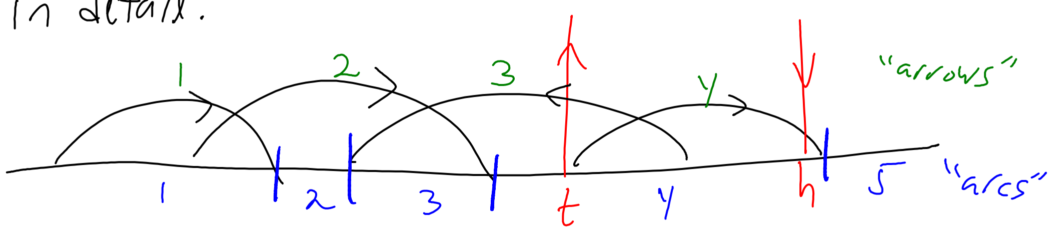
$$\begin{array}{cc} & \begin{array}{ccc} & h & \end{array} \\ \Lambda = \begin{array}{c}\text{tail@1}\\ \vdots\\ \text{tail@n+1}\end{array} & \left(\begin{array}{ccc|c} & \Lambda_h & & ? \end{array}\right) \\ & \begin{array}{ccc}\text{head@1} & \cdots & \text{head@n}\end{array}\end{array}$$

$$n\left\{\left(\overbrace{\underset{\text{(one per arrow row)}}{R}}^{n+1}\right)\right.$$

Q Given an $n\{(\underbrace{R}_{n+1})$ $\qquad R\Lambda_h = a_h$, $\ell_h\Lambda_h = 0$

$$\sum c_h \begin{pmatrix}R\\ \ell_h\end{pmatrix}^{-1}\begin{pmatrix}a_h\\ 0\end{pmatrix} \qquad \begin{pmatrix}R\\ \ell_h\end{pmatrix}\Lambda_h = \begin{pmatrix}a_h\\ 0\end{pmatrix}$$

Now in detail.



R was already written: $R = \overbrace{\left(\qquad\right)}^{n+1}$ } one row per arrow i; rows are relations

one col...

one col per arc α

Convention for red arrows $a_{\alpha i}$: The tail is in the middle of an arc. The head is just to the left of the head of a black arrow. $\Lambda = (a_{\alpha i})$ is $\left(\underbrace{\quad}_{n}\right)\}{n+1}$

Global relations: $(R\Lambda)_{ij} = \phi I_{ij}$ so $R\Lambda = \phi I$.

Per-head relations: $\Lambda_{\alpha(h_j), j} = D_R$.

$$\begin{array}{c|c|c|c} & a & b & c \\ \hline c & -1 & 1-X & X \\ & \alpha(h) & \alpha(t) & \alpha(h)+1 \end{array}$$

So writing $\Lambda = \left(\Lambda_1 \middle| \Lambda_2 \middle| \cdots \middle| \Lambda_n\right)$, we have

$$\begin{pmatrix} R \\ e_{\alpha(h_j)} \end{pmatrix} \Lambda_j = \begin{pmatrix} \phi e_j \\ D_R \end{pmatrix} \Rightarrow \Lambda_j = \begin{pmatrix} R \\ e_{\alpha(h_j)} \end{pmatrix}^{-1} \begin{pmatrix} \phi e_j \\ D_R \end{pmatrix}$$

Finally, our invariant is

$$\lambda = \sum_j s_j \Lambda_{\alpha(t_j), j} = \sum_j s_j \left(\begin{pmatrix} R \\ e_{\alpha(h_j)} \end{pmatrix}^{-1} \begin{pmatrix} \phi e_j \\ D_R \end{pmatrix} \right)_{\alpha(t_j)}$$

Analysis

Experimentally, in each individual summand the D_R term is just $-s_j D_R$.
(actually, this is easy to prove, seeing that $\det\begin{pmatrix} R \\ e_{\alpha(h_j)} \end{pmatrix}$ is Alex, and so is every last-row minor of that matrix).

$$\left(\begin{pmatrix} R \\ e_{\alpha(h_j)} \end{pmatrix}^{-1}\right)_{\alpha(t_j), j} = \underbrace{\left| R^{h_j}_{\hat{j}, \alpha(t_j)} \right|}_{R^{h_j}}$$

$$\begin{array}{r|c|c|c|} j & & R & \\ \hline & & & \\ \cdots & \bullet & \cdots & \cdots \\ & \alpha(h_j) & & \alpha(t_j) \end{array}$$

Aside $\begin{pmatrix} A & b \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}b \\ 0 & 1 \end{pmatrix}$

So, mod D_K,

$$\left(\begin{pmatrix} R \\ e_{\alpha(h_j)} \end{pmatrix}^{-1} \begin{pmatrix} \phi e_j \\ D_K \end{pmatrix} \right)_{\alpha(t_j)} = \phi \underbrace{e_{\alpha(t_j)}}_{\text{possibly shifted}}.$$

?

Aside. $M \in M_{n \times (n+1)}$, M_d is M with a dropped column, M_a is M with an added "standard" $(00..1..0)$ row. Why is

$$tr(M_d^{-1} \partial M_d) = tr(M_a^{-1} \partial M_a) \ ?$$

$$\| \text{above}$$

$$tr(A \partial A)$$

$$\| \text{above}$$

$$tr\left(\begin{pmatrix} A^{-1} & -A^{-1}b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \partial A & \partial b \\ 0 & 0 \end{pmatrix} \right) = tr(A^{-1} \partial A)$$