The first to do quandles was M. Takasak in 1942, about "kei" or "圭" ← chinese character

1. $a * a = a$
2. $(a*b)*b = a$ "involutive property"
3. $(a*b)*c = (a*c)*(b*c)$

Example If G is an Abelian group, set $a*b = 2b - a$. (call it $T(G)$)

Later came Joyce, Matveev, ... } published
1979 1982

Replace $2'$ by "$(\cdot * b)$ is invertible"

Definition "Rack": Just impose axioms 2 & 3

Conway - Cavin Wraith called it "Wrack" in 1959.

Scott Carter: "Shelf" is just axiom 3.

Let $(X, *)$ be a shelf. Set $C_n^R(X) = \mathbb{Z}X^n$ & $\partial^R : C_n \to C_{n-1}$ by

Thought (DBN) Is there a relationship between the following two diagrams:

$$\begin{array}{ccc} Q & \longrightarrow & QH \\ \downarrow & & \\ Leib & & \end{array} \qquad \begin{array}{ccc} \text{Knots} & \longrightarrow & KH, \ldots \\ \downarrow & & \\ A & & \end{array}$$

$$\partial^R(x_1, \ldots, x_n) = \sum^n (-)^i \big[(x_1 \ldots \hat{x}_i, \ldots, x_n)$$

This is formally the braidor complex?

$i=1$

$$-(x_1 * x_i, \ldots x_{i-1} * x_i, x_{i+1}, \ldots x_n)]$$

degenerate

Let $C_n^D \subset C_n^R$ be generated by $(x_1, \ldots x_n)$ s.t. for some i, $x_i = x_{i+1}$. Get

$$0 \longrightarrow C_n^D \longrightarrow C_n^R \longrightarrow C_n^Q \to 0$$

<u>Theorem</u> (Litherland-Nelson 2000)

$$H_n^R = H_n^D \oplus H_n^Q$$

<u>Theorem</u> (Przytycki, Niebrzydowski)

$H_3^Q(T(\mathbb{Z}/p)) = \mathbb{Z}/p$ + some similar results.