

The first to do quandles was M. Takasaka in 1942, about "kei" or "⊕" ← chinese character

1. $a * a = a$
2. $(a * b) * b = a$ "involution property"
3. $(a * b) * c = (a * c) * (b * c)$

Example If G is an Abelian group, set $a * b = 2b - a$.
(call it $T(G)$)

Later came Joyce, Matveev, ... } published 1979 } 1982

Replace 2' by "($\cdot * b$) is invertible"

Definition "Rack": just impose axioms 2 & 3

Conway - Gavin Wraith called it "Wrack" in 1959.

Scott Carter: "Shelf" is just axiom 3.

Let $(X, *)$ be a shelf.

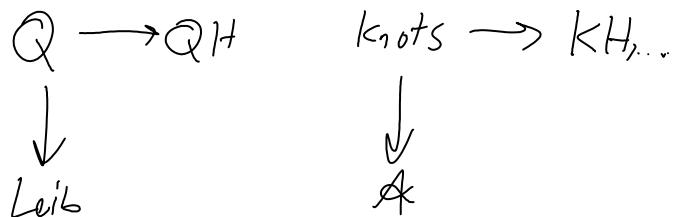
Set $C_n^R(X) = \mathbb{Z}\langle X^n \rangle$ &

$\partial^R: C_n \rightarrow C_{n-1}$

by

Thought (D&D) Is there a relationship

between the following two diagrams:



$$\partial^R(x_1, \dots, x_n) = \sum_{i=1}^n (-1)^i [(x_1, \dots, \hat{x}_i, \dots, x_n)]$$

This is formally the braid complex!

$$\text{degenerate} \quad i=1 \quad \downarrow \quad - (x_1 * x_i, \dots, x_{j-1} * x_i, x_{j+1}, \dots, x_n)$$

Let $C_n^D \subset C_n^R$ be generated by (x_1, \dots, x_n) s.t.

for some i , $x_i = x_{i+1}$. Get

$$0 \rightarrow C_n^D \rightarrow C_n^R \rightarrow C_n^Q \rightarrow 0$$

Theorem (Litherland-Nelson 2000)

$$H_n^R = H_n^D \oplus H_n^Q$$

Theorem (Przytycki, Niebrzydowski)

$$H_3^Q(T(\mathbb{Z}/p)) = \mathbb{Z}/p \quad + \text{some similar results.}$$