# Vgl

June-26-10
4:41 AM

Throughout, a "poset" $X$ means either an ordinary poset, or the special poset "$\Rightarrow\Leftarrow$", or "contradiction". If $a,b\in X$, let $X_{a<b}$ be the poset $X$ with the relation $a<b$ added (this may be a contradiction!). Likewise for $X_{a=b}$ and $X_{a>b}$. We will always allow $\Rightarrow\Leftarrow_{a<b}$ (etc.), and this is always $\Rightarrow\Leftarrow$.

[The treatment of $\Rightarrow\Leftarrow$ is not as good as it should be]

Vgl is the algebra generated as a vector space by pairs $[X,w]$, where $w\in U(gl(X))$, modulo the relations

1. Linearity in $w$.
2. $[\Rightarrow\Leftarrow, w]=0$ for all $w$.
3. $[X,w]=[X_{a<b},w]+[X_{a=b},w]+[X_{a>b},w]$
4. If $f:X\to Y$ is an isomorphism of posets, then $[X,w]=[Y,f_*w]$

The product on Vgl is

$$[X,w_1]\cdot[Y,w_2]=[X\cup Y, w_1w_2]$$

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Question What are the co-invariants of Vgl? Or maybe, what are the characters of Vgl?

("co-invariants" are in some "internal" sense, which needs to be made explicit).

It is not hard to construct characters corresponding to partitions.

1. Can I make this book natural?
2. Can I prove that this is all?