Furusho on Racinet

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From Furusho's arXiv:0808.0319v2:

APPENDIX A. REVIEW OF THE PROOF OF RACINET'S THEOREM

In [R] theorem I, Racinet shows a highly non-trivial result that DMR_0 is closed by the multiplication (0.1). However his proof looks too complicated. The aim of this appendix is to review the essential part ([R] proposition 4.A.i) of his proof clearly in the case of $\Gamma = \{1\}$ in order to help the readers to catch his arguments.

this is exactly what an automorphism-group interpretation ought to make manageable.

In [R]3.3.1, $\mathfrak{dmr}_0$ is introduced to be the set of formal Lie series $\psi \in \mathfrak{F}_2 = \{\psi \in U\mathfrak{F}_2 | \Delta(\psi) = 1 \otimes \psi + \psi \otimes 1\}$ satisfying $c_{X_0}(\psi) = c_{X_1}(\psi) = 0$ and

$$\Delta_*(\psi_*) = 1 \otimes \psi_* + \psi_* \otimes 1 \tag{A.1}$$

with $\psi_* = \psi_{\mathrm{corr}} + \pi_Y(\psi)$ and $\psi_{\mathrm{corr}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} c_{X_0^{n-1}X_1}(\psi) Y_1^n$. It is the tangent vector space at the origin of DMR_0.

Theorem A.1 ([R] proposition 4.A.i). *The set $\mathfrak{dmr}_0$ has a structure of Lie algebra with the Lie bracket [6] given by*

$$\{\psi_1, \psi_2\} = d_{\psi_2}(\psi_1) - d_{\psi_1}(\psi_2) - [\psi_1, \psi_2]$$

where d_ψ is the derivation given by $d_\psi(X_0) = 0$ and $d_\psi(X_1) = [X_1, \psi]$.

Ain't it the case that dmr_0 is the $(0, a_2)$ subalgebra of tder_2? (with $a_2 \leftrightarrow \psi$)

Question: What is dmr_0 as a Lie algebra? Is it a free Lie algebra? Is it the free Lie algebra on two generators?

Question: Is there a "centre of gravity" argument implying that dmr_0 is a direct summand of tder_2? Maybe

$$\mathrm{tder}_2 \cong \mathrm{Lie}_2 \oplus \mathrm{dmr}_0 \quad \left(\text{as Lie algs, not just as v.s.}\right)?$$

Question: Topologically speaking, what are inner derivations as a subspace of tder?

Question: What is the tangent space to the space of "alternative Lie brackets" on Lie_n, near the standard "alternative Lie brackets" on Lie_n, near the standard bracket? When should two "alternative brackets" be regarded as equivalent up to an automorphism?