<u>Bezout's Theorem</u> If C & D are smooth plane curves $C, D \subseteq \mathbb{P}^2$ which meet transversely, then

$$\#(C \cap D) = \deg(C) \cdot \deg(D);$$

in general

$$\deg C \cdot \deg D = \sum_{p \in C \cap D} IM_p(C, D)$$

Define IM: $IM_p(C,D) = \dim_{\mathbb{C}} \mathcal{O}_{C,p} \otimes_{\mathcal{O}_{\mathbb{P}^2,p}} \mathcal{O}_{D,p}$

In general, $C, D \subset \mathbb{P}^n$, $\dim C + \dim D = n$, and if $C \cap D$ is transverse, then $\deg C \cdot \deg D = \#(C \cap D)$.

In general,

$$\deg C \cdot \deg D \neq \sum_{p \in C \cap D} IM_p(C, D)$$

Serre's intersection formula:

$$IM_p(C,D) = \sum_i (-1)^i \dim \mathrm{Tor}_i^{\mathcal{O}_{\mathbb{P}^n,p}}(\mathcal{O}_{C,p}, \mathcal{O}_{D,p})$$

$$= \dim(\mathcal{O}_C \otimes \mathcal{O}_D) + \text{error terms.}$$

determined by $C \cap D$ as schemes   higher.

<u>Goal</u> Interpret $C \cap D$ in a way that remembers $IM_p(C, D)$

$$\begin{pmatrix}\text{Algebraic} \\ \text{topology}\end{pmatrix} = \begin{pmatrix}\text{presenting sets by} \\ \text{generators and relations (and more)}\end{pmatrix}$$

<u>Idea</u> Replace commutative rings by topological rings.

[up to homotopy, so all v.s. are the same, and non-discrete]

Serre's Intersection Formula:

$$IM_p(C,D) = \sum_i (-1)^i \dim_{\mathbb{C}} \pi_i \left(\mathcal{O}_{C,p} \otimes^L_{\mathcal{O}_{\mathbb{P}^2,p}} \mathcal{O}_{D,p} \right)$$

<u>Def</u>: A <u>derived</u> scheme is a top space X with a sheaf $\mathcal{O}_X$ of top. commutative rings, with $(X, \mathcal{O}_X) \overset{\text{locally}}{\cong} \operatorname{Spec} A$, where A is a top. commutative ring

.... everything has to be given a meaning.