

Divided powers: $e^{(r)} := e^r / r!$ $f^{(r)} := f^r / r!$

$U(\mathfrak{sl}_2)$ is the \mathbb{Z} -submodule of $U(\mathfrak{sl}_2)$
generated by $e^{(r)}, f^{(r)}$ & h .

Categorical \mathfrak{sl}_2 actions

Data: Graded additive categories $\mathcal{O}(\lambda)$
with functor

$$\begin{aligned} E : \mathcal{O}(\lambda) &\rightarrow \mathcal{O}(\lambda+2) \\ F : \mathcal{O}(\lambda+2) &\rightarrow \mathcal{O}(\lambda) \end{aligned} \quad \left[\begin{array}{l} \text{also} \\ E^{(r)} \& F^{(r)} \end{array} \right]$$

So that

$$EF|_{\mathcal{O}(\lambda)} \cong FE|_{\mathcal{O}(\lambda)} \oplus Id[\lambda-1] \oplus Id[\lambda-3] \oplus \dots \oplus Id[-\lambda+1]$$

if $\lambda > 0$, & opposite if $\lambda < 0$.

$$\forall E^{(r)} \circ E \cong E^{(r+1)}[-r] \oplus \dots \oplus E^{(r+1)}[+r]$$

3:30