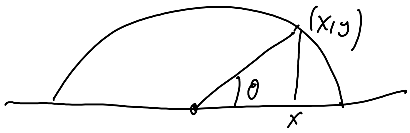
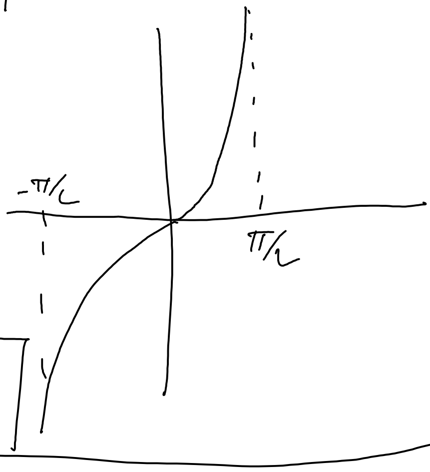
The Formulas

March-11-09
8:52 AM

$\theta' = \sin\theta \qquad \frac{d\theta}{dt} = \sin\theta$



$$\frac{d\theta}{\sin\theta} = dt$$



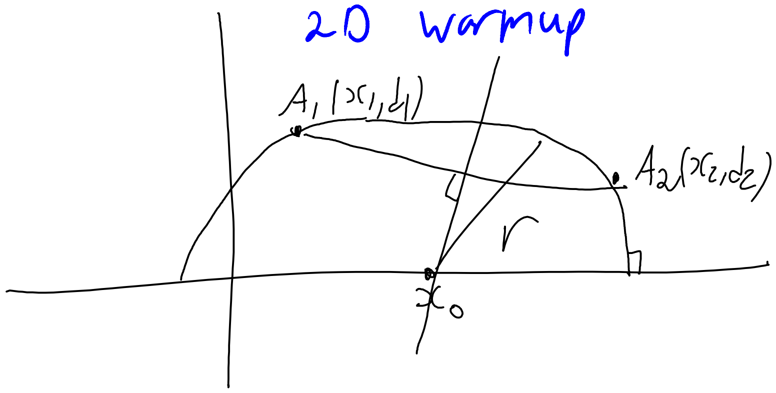
$\Rightarrow$ In[1]:= `DSolve[{θ'[t] == Sin[θ[t]], θ[0] == π/2}, θ, t]`

Out[1]= `{{θ → Function[{t}, 2 ArcCot[e^-t]]}}`

$\Rightarrow \theta = 2\arctan(e^t)$

Thus $t = \log\tan\theta/2$

2D warmup



Q Given A_1 & A_2, find x_0 & r.

The case $x_1 = x_2$ must be treated separately!

$$(x_1 - x_0)^2 + d_1^2 = (x_2 - x_0)^2 + d_2^2$$

$$x_1^2 + d_1^2 - 2x_1x_0 = x_2^2 + d_2^2 - 2x_2x_0$$

$$x_1^2 + d_1^2 - x_2^2 - d_2^2 = 2(x_1 - x_2)x_0$$

$$x_0 = \frac{x_1 + x_2}{2} + \frac{d_1^2 - d_2^2}{2(x_1 - x_2)}$$

Q Given $A_1(x_1, y_1, d_1)$ and $A_2(x_2, y_2, d_2)$, find $A_3(x_3, y_3, d_3)$ which is "1/n of the way from A_1 towards A_2.

Sol'n If $(x_1, y_1) = (x_2, y_2)$, just zoom. Otherwise,

1. Translate/rotate so that $(x_2, y_2) \to (0,0)$
   $(x_1, y_1) \to (x_1', 0)$

meaning, find α s.t. $\begin{pmatrix} \cos\alpha & \sin\alpha \\ \ldots \end{pmatrix}\begin{pmatrix} x_1 - x_2 \\ \ldots \end{pmatrix} = \begin{pmatrix} x_1' \\ \ldots \end{pmatrix}$

using $\cos\alpha$ ($y_1 - y_2$) (0)

Really, just find $c = \cos\alpha$ & $s = \sin\alpha$, using

$$\begin{pmatrix} c \\ s \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix} \Big/ \| \cdot \|$$

2. Find x_0 & r as in the 2D warmup:

$$x_0 = \frac{x_1'}{2} + \frac{d_1^2 - d_2^2}{2x_1'} \qquad r = \left((x_1' - x_0)^2 + d_1^2 \right)^{1/2}$$

3. Translate / Scale so that $x_0 \to 0$, $r \to 1$:

$$x_1'' := (x_1' - x_0)/r$$

$$x_2'' := (-x_0)/r$$

4. Find $\theta_{1,2}$: $\quad \theta_i := \arccos x_i''$ $\quad$ (make sure $\theta_i \in [0, \pi]$)

5. Find $t_{1,2}$: $\quad t_i = \log\tan \theta_i/2$

6. set $t_3 = t_1 + \frac{1}{n}(t_2 - t_1)$

And then go backwards $5 \to 4 \to 3 \to 2 \to 1$ with t_3