Sniatycki: Commutation of Quantization and Reduction

April-17-09
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(P, ω) a symplectic manifold.

Prequantization condition: $[\omega] \in H^2(\mathbb{Z}, P)$

Find a line bundle $L \to P$ with connection ∇

so that $\mathrm{curv}(\nabla) = \omega$.

Find a ∇-flat $\langle \cdot | \cdot \rangle : L \times L \to \mathbb{C}$.

$$\mathcal{P} : C^\infty(P) \times S^\infty(L) \longrightarrow S^\infty(L)$$

$$(f, \sigma) \longmapsto \mathcal{P}_f \sigma := \left(-i\hbar \nabla_{X_f} + f\right)\sigma$$

$$[\mathcal{P}_{f_1}, \mathcal{P}_{f_2}] = i\hbar \mathcal{P}_{\{f_1, f_2\}}$$

Polarization $F \subset TP \otimes \mathbb{C}$

involutive, Lagrangian

$$S_F^\infty(L) := \{\sigma \in S^\infty(L) \mid \nabla_F \sigma = 0\}$$

$$C_F^\infty(P) := \{f \in C^\infty(P) \mid [X_f, F] \subset F\}$$

$$Q_f = \mathcal{P}_f \Big|_{C_F^\infty(P) \times S_F^\infty(L)}$$

Unitarization If $F \oplus \bar{F} = TP \otimes \mathbb{C}$ and

$i\omega(\bar{u}, u) \geq 0$ for all $u \in F$.

set $\mathcal{H}_F$ = Sections in $S_F^\infty(L)$ that are square integrable.

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