Rephrasing KV-Naive

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<u>KV-Naive:</u> If $f, g \in \mathrm{Fun}(G)^G$ and $(\Phi f) \in \mathrm{Fun}(\mathfrak{g})^G$ is defined by $(\Phi f)(x) = j^{1/2}(x) f(e^x)$ with j as usual, then

$$\underset{\substack{\uparrow \\ \text{convolution} \\ \text{on } G}}{\Phi(f * g)} = \Phi(f) \underset{\substack{\uparrow \\ \text{convolution} \\ \text{on } \mathfrak{g}}}{*} \Phi(g)$$

$\boxed{j(x) = \text{jacobian of } x \mapsto e^x}$

Rephrasing goals: 1. Replace "convolution" by multiplication of $\mathcal{U}(\mathfrak{g})$-valued measures.
(that is, $w \in \mathrm{Fun}(\mathfrak{g}) \longmapsto \int w(x) e^x dx \in \mathcal{U}(\mathfrak{g})$)

2. The "inputs" f, g should be replaced by functions on $\mathfrak{g}$.

KV-Rephrased: If $u, v \in \mathrm{Fun}(\mathfrak{g})^G$ then

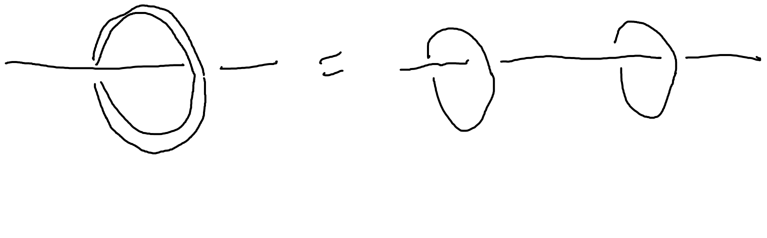
$$\int j(x)dx\, j(y)dy\, j^{-1/2}(x)u(x)\, j^{-1/2}(y)v(y) \cdot e^x e^y \cdot \underbrace{j^{1/2}(\log e^x e^y)}_{j^{-1/2}\ ?} = \int dx\,dy\, u(x)u(y)\, e^{x+y}$$

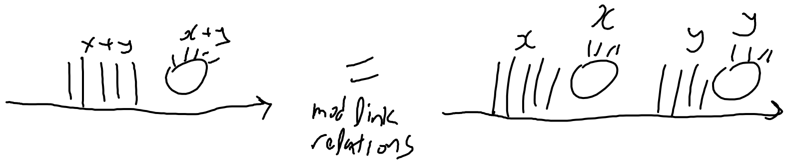
$u(x) = j^{1/2}(x) f(e^x)$
$v(x) = j^{1/2}(x) g(e^x)$
So
$f(e^x) = j^{-1/2} u(x) \ldots$

$$\begin{tikzcd} & \mathfrak{g} \arrow[dl, "\exp"'] \arrow[d, "\exp"] \\ G \arrow[r, leftrightarrow] & \mathcal{U}(\mathfrak{g}) \end{tikzcd}$$

$$\underset{\substack{\text{The "group ring"} \\ \text{of } \mathfrak{g}}}{S(\mathfrak{g})} \xrightarrow{\chi} \underset{\substack{\text{The group ring} \\ \text{of } G}}{\mathcal{U}(\mathfrak{g})}$$

<u>Claim</u> when restricted to invariants and "wheels", this is an algebra isomorphism.

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