Glasgow Talk

February-10-09
9:36 AM

More at http://katlas.math.toronto.edu/drorbn/AcademicPensieve/2008-10/index.html

Title: Convolutions on Lie Groups and Lie Algebras and Ribbon 2-Knots

Abstract: I'll start my talk by describing the Kashiwara-Vergne Conjecture (19??, Proven Alekseev-Meinrenken, 2006), which states that for any finite dimensional Lie group, certain convolutions on the group are equal to certain convolutions on its Lie algebra, and I'll end my talk in knot theory. Going backward from the hard to the easy, here's the whole thing in one paragraph:

Convolutions are integrals so we need to prove an equality of integrals. This we do (roughly) by finding a Measure Preserving Transformation (MPT) which carries one integrand to the other. This has to work for any Lie group, so the MPT better be given as a universal formula. At this generality all we have to work with is the Lie bracket, and bracket-made formulas can be pictured as certain trivalent diagrams (which in themselves are subject to the Jacobi, or "IHX" relation). Thus we are really seeking a certain big sum F of trivalent diagrams, which, in order to describe our desired MPT, must satisfy certain equations mod IHX. Our space A^w of trivalent diagrams turns out to be the "associated graded" space of the space K^w of ribbon 2-knots in 4-space, and the equations F needs to solve are the equations one needs to solve to get a well-behaved "expansion" Z:K^w--->A^w.

There's still too much interplay between MPT's & differential operators; that is, between measures and functions.

Thus the Kashiwara-Vergne Conjecture, itself a witness to the famed "orbit method", is more or less the same as a natural problem in knot theory, and since knot theory is related to associators and to quantum groups, so is the Kashiwara-Vergne Conjecture. Cool, eh?

~~Oh well, there are still gaps in the story. I hope to at least explain to you why I am convinced it is nevertheless true (perhaps modulo small corrections) and I hope to explain the main gap - with so many smart people present, surely someone will be able to help me.~~

Rephrased in http://www.math.toronto.edu/~drorbn/Talks/Glasgow-0904/

Structure of the talk:

Act I. Prologue: Introducing the characters.

1. Ribbon topology in $\mathbb{R}^4$.

2. Kashiwara-Vergne and the orbit method.

Act II Dialogue: An unlikely encounter between near strangers.

Act III Epilogue: They will live happily ever after, yet their story is yet to be told.

Further Directions:

1. complete details.
2. Use to study associators.
3. Relate to BF theory.
4. Extend to knotted simplicial spaces.
5. Extend to virtual knots, general bi-algebras, quantum groups, Etingof-Kazhdan, unknown topology and unknown physics.